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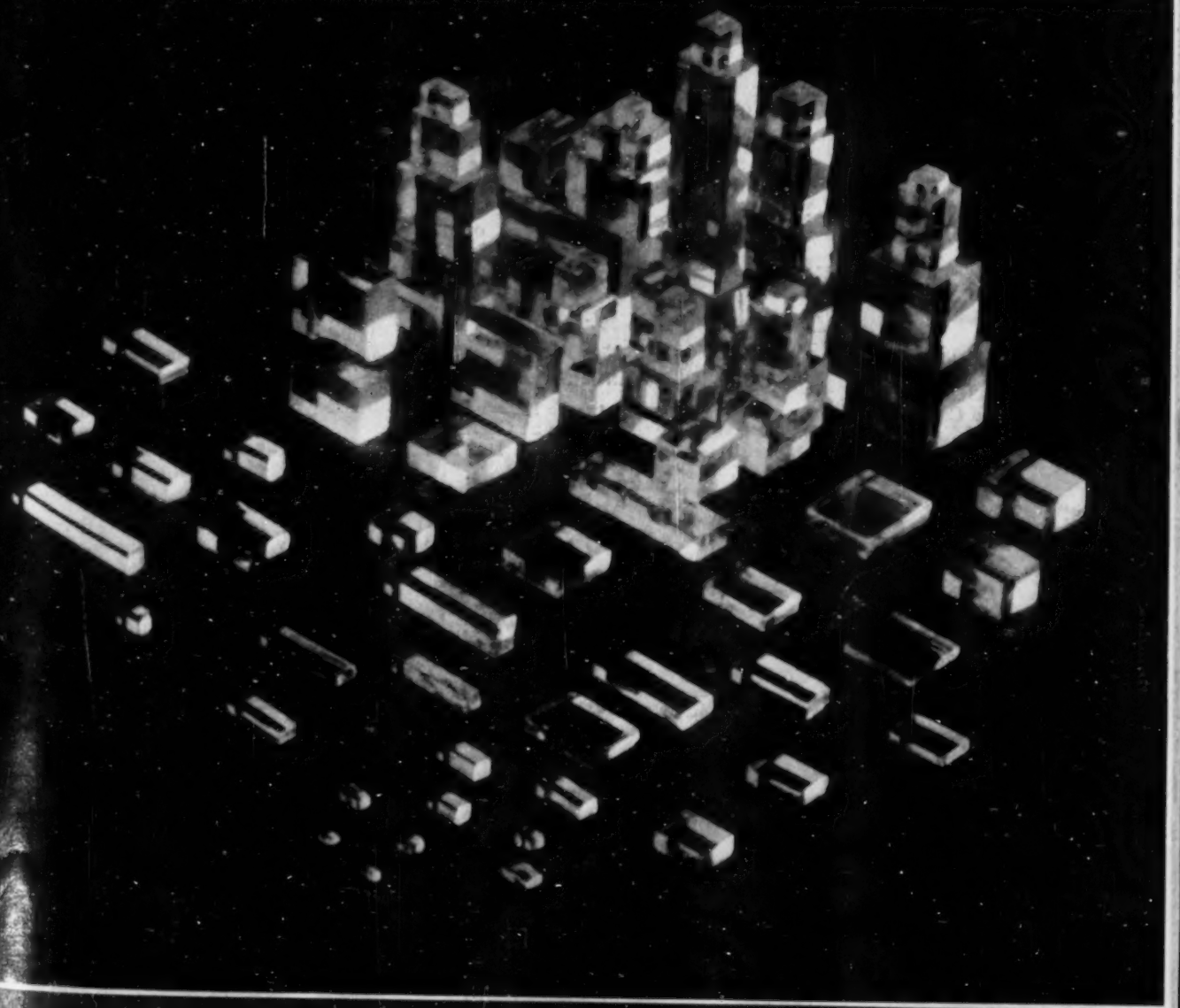

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JOURNAL OF APPLIED
PHYSICS

FEBRUARY, 1939 Vol. 10, No. 2

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The Question Before Intelligent Individuals

IF there is one thing that physicists have learned during the past few years, it is the great possibility of error involved in extrapolating the laws of nature beyond our physical experience. The intellectual upheavals caused by the introduction of the uncertainty principle, the new quantum mechanics and the new types of statistics have made the physicist wary of his faith in principles which are "as old as the hills." A new critical and questioning attitude has developed which perhaps above all else accounts for the almost unbelievably rapid development of physics in the past decade.

Shall this same attitude be assumed in studies of our social and economic structures? An emphatic affirmative answer to this question was given by Professor P. W. Bridgman in an address on "Society and the Intelligent Physicist"[1] at the Washington meeting of the American Association of Physics Teachers and in his book *The Intelligent Individual and Society*.[2] Professor Bridgman pleads that we face courageously our age-old social laws and customs with the same critical questioning attitude and the same intellectual honesty with which we have faced our beliefs concerning natural laws.

For instance, is it true that "all men are created equal"? Do all men have the same loyalties, the same sense of responsibilities and the same drives to do good? Can a stable and good social structure be built without taking into account individual differences? Do words like "duty," "ought" and "should" have meanings of a kind different from physical quantities (such as the position and size of an electron) which have meanings only in terms of the operations by which they are measured.

Professor Bridgman wonders whether the assumptions at the foundations of the present social structure can survive intelligent scrutiny or whether the present structure would be stable if every individual acts intelligently. Many of our present social laws are based upon mysticism or upon the thesis that a man will behave naturally in a way that is good for all. In substance Professor Bridgman asks, "Are we being intelligent in believing that the social structure we have built will bring maximum good for the maximum number in face of the experimental evidence which dictators are providing us today of the excesses which may be committed under present systems?" These are only a few of the questions he raises.

Professor Bridgman offers us a challenge. Whether it is accepted or not only the future can tell.

[1] Will probably be published in the April, 1939 issue of the *American Physics Teacher*.
[2] Macmillan Company, New York, New York.

Cultural Values of Physics*

BY DAVID DIETZ

Science Editor, Scripps-Howard Newspapers, Cleveland, Ohio

I

THERE was a time when the classics formed the cultural bond that united educated men throughout the world. Every high-school boy read his Caesar, Cicero and Virgil in Latin, his Xenophon and Homer in Greek. He continued in college with Horace and the other Latin poets and made the acquaintance of the Greek dramatists and philosophers. Thus all educated men were bound together by a common educational experience and their thinking was rooted in a common source of inspiration, the classical learning of ancient Rome and Greece.

With the rise of the twentieth century, the classics lost their hold. I do not suppose that there is a college in America which lists a knowledge of Greek among its entrance requirements. Many colleges, including, of course, the scientific schools, do not require Latin.

The dethronement of the classics is so nearly complete today that it is doubtful if the young student, stepping upon a college campus for the first time, realizes the change which has taken place in the structure of education. I carry a sharp picture of the changing process since it was my interesting fortune to live through it.

The decline of the classics has, unquestionably, been a by-product of the rise of science. Many educators, however, have lamented the fact that there is no longer a common cultural tie among learned men. For with the rise of science has come the rise of specialties and with the rise of specialties has come a division of tongues. Each specialist speaks a language of his own and this fact has sometimes been a handicap to understanding and a stumbling block to progress.

It is inevitable that each specialist must pursue his own line of attack further and further into the frontier of the unknown. It is, therefore, equally inevitable that these pioneers must draw away further and further from each other. But this makes it all the more essential that there be some common bond of understanding both for them and for all educated men in general.

Since this is pre-eminently an age of science, an age chiefly distinguished for its advances in scientific understanding and engineering achievement, it is obvious that such a common basis must be found in the realm of science. I am firmly convinced that there is only one subject that can furnish this necessary foundation for modern education. It is the subject of physics. I wish to urge, therefore, that educators everywhere give thought to ways of shaping educational policies to bring about this united emphasis upon the teaching of physics.

II

Physics derives its first great cultural value from the fact that the present age cannot be understood without an understanding of physics. Physics is the foundation stone of the age in which we live. It was ushered in by discoveries in the realm of physics.

It was the announcement of the x-ray by Roentgen in 1895, followed in quick succession by Becquerel's disclosure of radioactivity, the isolation of radium by the Curies, and the researches into the nature of the atom and the electronic theory of matter by Thomson, Lorentz, Rutherford, Soddy, and others, which ushered in the magnificent edifice of twentieth century science. These discoveries not only set the pace but furnished the foundation for the century's growth. In 1900 the electron was a theory. In the next decade, Dr. Millikan was to perform his famous experiments to measure the electric charge upon the electron, experiments destined to win the Nobel prize in physics. Today, the world has put the electron to work. In the

* Address given at the annual meeting of The Pennsylvania Conference of College Physics Teachers, Pennsylvania State College, October 14, 1938.

vacuum tubes of our radio sets, in the photo-electric cell, in other electronic tubes and in the x-ray tube, we are making daily use of the electron.[1]

It will be seen, therefore, that only through a knowledge of physics can the student gain the historical perspective needed for an understanding of the temper and the tempo of the age in which we live.

But we need an understanding of physics just as much for the comprehension of the individual details which make up the picture of our modern age. Without it, the radio set is a complete riddle, the gasoline engine becomes a baffling puzzle, the electric light, a mystery without explanation. We can only understand these things and the countless other mechanical and electrical marvels around us with the aid of the principles of physics. And this fact brings me to another great cultural value of physics.

Physics derives its second great cultural value from the fact that it is the foundation of all the sciences. This has not always been realized for many reasons.

In the first place, a trick of language alienated physics from its offspring and obscured the connection. Applied physics became known as engineering and the applications of physics were disseminated under the names of mechanical engineering, civil engineering, metallurgy, electrical engineering, and so on. But let us not forget that it was the physicist who launched each of these specialties. It was Galileo who laid down the fundamental laws of the machine. Newton's mathematical expression of Galileo's dictum, force equals mass times acceleration, is the foundation of every machine in the world. Similarly, electrical engineering grew from the experiments of Oersted, Ampère, Faraday, and Ohm. The first principles of every engineering science are the laws of physics and no real understanding of engineering advances is possible without a knowledge of physics.

The fundamental position of physics as the foundation stone of all science was missed, in the second place, because in certain branches of science this relationship was not at first clear. It took time to bring the development of atomic theory to the point where it was plain that chemical phenomena represented the operation of physical laws. Today we see the relationship of chemistry to physics clearly expressed by the use of the term, "physical chemistry" which, perhaps, might just as well be written "chemical physics." The chemist invokes the laws of physics to explain the behavior of atoms and often we find both chemists and physicists working upon identical problems.

Biology, long regarded as a thing apart, has been brought into the fold of the physical sciences. This recognition of relationship was first celebrated with the creation of "biochemistry," a field of research which has been unusually fruitful. More recently this has been extended into "biophysics." Not long ago I was talking to the director of an important medical laboratory in the Middle West. "You will be surprised to know the latest addition to my staff," he said. "It is a full-time physicist."

This new demand for well-trained mathematical and experimental physicists in many laboratories outside the realm of physics, is one of the most interesting and important trends of our times. Not only are chemical, biological, and medical laboratories seeking the aid and services of physicists, but many industrial laboratories are discovering that chemists and engineers are not sufficient to deal with the intricate problems met today in mining, metallurgy, electrical design and construction, and many other fields. This trend was discussed at some length at the Conference on Industrial Physics arranged by the Physics Department of the University of Pittsburgh on November 15, 1935.[2] I had the honor of appearing on that program[3] and I remember well the discussions there, particularly the paper by Dr. A. W. Hull of the General Electric Company upon "Putting Physics to Work."[4]

The physicist is doing his share to usher in the new world of taller buildings, longer bridges, swifter trains, safer aircraft, and finer homes, a new world of greater beauty, deeper comfort and smoother efficiency. He is helping to build this new world out of stronger steels, tougher alloys, better aluminum products, more useful plastics, harder abrasives, more powerful machine tools, and more ingenious automatic controls.

III

A third great cultural value of physics is derived from the importance of physics for the future. I like to talk about the future for I believe that we should face the future with courage and confidence. I believe that the world will move within the next century into a period far more remarkable than the present, and I believe that the leadership for this great progress will come from America.

I have spoken of the taller buildings, the longer bridges, and the faster trains and airplanes that are now evolving. But no bigger mistake could be made than to think of the future as nothing more than an exaggerated picture of the present.

This is the mistake, as the British satirist and caricaturist, Max Beerbohm has pointed out, that every century has made. The sixteenth century thought that the seventeenth century would be only a magnified picture of itself. The seventeenth thought the same of the eighteenth, and the eighteenth thought likewise of the nineteenth. Yet each century had a personality and a development all its own.

How smug the nineteenth century was in this regard can best be described by borrowing a story from Dr. Millikan. He tells how, as a student in Europe, he attended the annual session of the British Association for the Advancement of Science in 1893. An eminent British physicist rose to address that august assembly and spent his time giving thanks that he had lived at the close of the nineteenth century.

For, he said, the nineteenth century had seen the completion of the great edifice of physics. All the laws of nature had been discovered and catalogued. Nothing remained for the physicists of the future but to repeat the experiments of the past. Perhaps some twentieth century physicist might carry to four decimal places a determination which the nineteenth century physicist had left at three.

And how quickly that smug view of nature was overturned! Just two years later, in 1895, Roentgen showed the German Physical Society the world's first x-ray pictures. Those pictures—so commonplace today, so startling then, the picture of coins and keys showing through the leather walls of a purse, of bones showing through the skin and flesh of a human hand—those pictures were proof that far from completing the cataloguing of nature's laws, the nineteenth century physicist had only made a beginning.

In thinking of the twenty-first century, I would ask you not to look at those things which represent the most complete accomplishments of our present day but to look at those things which we are just now beginning to comprehend. I would ask you to visit the laboratories and study the researches under way rather than to visit the factory or the market place to study the finished achievements.

Sometimes these laboratory experiments look confused and useless, but let us not fool ourselves. I am reminded of the story of the visit which the prime minister of England paid to Faraday's laboratory at the Royal Institution in London. Faraday was then engaged in those experiments upon the laws of electricity, experiments in physics, if you please, from which have come every electric generator, transformer, and motor in the world.

"What's the use of all this?" the prime minister asked Faraday.

"Don't worry, milord," Faraday is said to have replied, "you'll tax it yet."

When we recall all the taxes paid today by the electrical industry, and all the taxes we help pay when we pay our electric light bills, we are inclined to agree with Faraday.

And I am reminded of another story, this one about our own great statesman, patriot, and physicist, Benjamin Franklin. That worthy did many things his neighbors did not altogether understand, like flying kites, for example. One day, a neighbor woman asked Ben the very same question that the prime minister had asked Faraday. "Ben," she said, "what's the use of all this?" And Franklin, being a good Yankee, replied in Yankee fashion with another question. "What's the use of a baby?" he asked.

We all know the answer to that question. A baby can grow up to be a very useful man or woman, and when we see the veritable giant into which the baby electricity has grown, we realize the wisdom of Franklin.

Now when you attempt to picture the future I want you to give thought to some of the babies of the physical laboratory. They will grow up to make the twenty-first century a personality in its own right, different from the century we know.

I think first of the "babies" in the field of the production of power. The age in which we live rests upon a foundation of physical power. Imagine, for a moment, what would happen to our great cities, if electric power and the power of the gasoline engine were suddenly removed. Every electric light would go out. Every electric motor would stop. Every automobile would become useless.

Physical power is the basis of economic and national power. Professor Leith of the University of Wisconsin, has pointed out that it is often said that the World War transferred world power from Great Britain to America. But he believes that the World War only made evident what had already happened. In the nineteenth century, Great Britain produced half of the world's physical power. Today, the output of energy in the United States from coal, oil, natural gas and water power amounts to half of the world's total. It is this fact, rather than the last war, Professor Leith believes, that explains the dominant position of America in the world of today.[5]

Any new source of energy, therefore, will be of supreme importance to the future and a cheap and abundant source of energy will change the shape of the future in ways which we can only attempt to guess. As is well known to you, attempts are being made to find such sources of power.

It is being sought, first of all, in attempts to put the sun to work. It would not be fair to say that the sun's energy goes to waste, since its light and heat makes life upon this earth possible. But it is perfectly true that we waste the greater part of the energy which the sun sends us. It has been calculated that the amount of energy falling upon every square yard of earth's surface per second is the equivalent of one and one-third horsepower.

Physicists have long dreamed of putting this solar energy to work. One of the first suggestions made was to concentrate the sun's heat by means of mirrors upon a steam boiler. Dr. C. G. Abbot, secretary of the Smithsonian Institution, has been a pioneer in this field, and his solar engine,[6] which has been exhibited at meetings of the American Association for the Advancement of Science, at the Great Lakes Exhibition in Cleveland, and elsewhere, is well known to many of you. Dr. Abbot incorporated many ingenious features in his device. A parabolic mirror concentrates sunlight upon the boiler which is actually two concentric glass tubes with a vacuum between them. Thus while there is only a slight barrier to the entrance of the sun's radiant energy, there is a considerable barrier against the loss of heat by atmospheric conduction. Steam is generated in the inner tube upon the flash boiler principle.

Another method for the utilization of solar energy which seems highly promising to many scientists is the conversion of sunlight into electricity by photoelectric methods.

About a year ago, on a visit to the research laboratories of the Westinghouse Electric & Manufacturing Company at East Pittsburgh, I was shown four photoelectric cells like those used in light meters which had been connected to a toy electric motor, the sort you might buy for a small boy at Christmas. When sunlight fell upon the cells, enough electricity was generated to run the motor. The research men referred to the motor laughingly as "a one fly-power motor." But again, it must be remembered that we were looking at a scientific baby.

Perhaps the day will come when our houses will be roofed with photoelectric cells instead of shingles and we will make electricity, instead of hay, while the sun shines.

It is of the utmost significance that during the present year both Harvard University and Massachusetts Institute of Technology were given funds totaling about $1,000,000 by Dr. Godfrey L. Cabot to investigate the problem of solar energy. M.I.T. is to concentrate upon the direct utilization of solar energy by such means as I have been describing while Harvard is to investigate photosynthesis, the method by which plants utilize solar energy.

Another direction in which many scientists are looking for a new source of energy is the interior of the atom. That the conversion of matter into

energy would release tremendous stores of power was shown as early as 1905 when Albert Einstein wrote his equations for the inertia of energy. Experiments in artificial radioactivity during the last five years have confirmed these equations of Einstein.

As you know, it has been calculated that the atomic energy in a glass of water would be enough to drive an ocean liner from New York to Cherbourg and back again. It is easy to see how different a world this would be if, instead of filling your automobile tank with gasoline every other day, you merely filled it with water once a year.

Second in importance to power in this world of ours, is the possession of raw materials. Here again we are fortunate to be citizens of America since this nation is the largest owner, producer and consumer of minerals, leading in the production of iron, copper, lead, zinc, aluminum, phosphates, gypsum, and sulphur.[7]

But the world's greatest reservoir of minerals is the ocean. A new world would dawn if we once learned to mine the ocean successfully. This is not as wild an idea as it sounds, for, as many of you know, a successful beginning has already been made.

If you use ethyl gasoline in your automobile, the chances are that it was made with bromine that was mined from the ocean. At Kure Beach near Wilmington, North Carolina, is the plant of the Ethyl-Dow Chemical Company. Sea water is pumped through the plant and bromine extracted from it by a relatively simple chemical process.

During the course of a year, the two giant electrically-driven centrifugal pumps lift about a square mile of ocean, 80 feet deep, into the towers of the Ethyl-Dow plant at Kure Beach. Chemists of the plant calculate that while they got the bromine out of that volume of sea water they missed about $96,379,460 worth of mineral wealth. Included in it was $29,300 worth of silver and $42,000 worth of gold.

While gold and silver appeal to our imaginations, a far greater amount of wealth was in that sea water in the form of sodium chloride, epsom salts, calcium chloride, potassium chloride, aluminum, magnesium, strontium carbonate, iron, copper, and iodine.

From the physical laboratory may come techniques in the future which will extend the ability of man to mine the ocean. And it takes little imagination to perceive how profoundly this will change the trend of civilization.

I have alluded to the new studies at Harvard upon photosynthesis. Perhaps some day, as Dr. Slosson once suggested, we may know as much chemistry as a tree. Perhaps we should say as much physics as a tree. When that day comes, artificial photosynthesis will be possible, and perhaps we will solve the farm problem by abolishing the farm and the cycle of the soil and by manufacturing our food in factories run by sunlight.

We may also expect great changes in the future from the application of physics to biology and medicine. Recently, as some of you know, I undertook to survey the field of medicine in my book, *Medical Magic*. I devoted the last chapter of the book to a glance at the future and in it I wrote:

"Of one thing we can be certain: that every advance in chemistry and physics, every new step in the understanding of the behavior of the molecule, the atom, and the electron, will have its influence upon medical progress. Already the medical laboratories of the world are making use of all the knowledge that physics and chemistry has to offer."[8]

IV

An example of the application of the technique of physics to biology is the development of the so-called "brain-wave machine" in which vacuum tube amplifiers are used to amplify the electrical currents generated in the brain. A new concept of brain activity and a new understanding of the nerve cell, its functions, and its behavior, are coming from these studies.

In the study of the potent drugs of life, the hormones, the vitamins, the enzymes, and the other important chemical factors, the constant attempt is to isolate them in pure crystalline form so that they may be studied with all the resources of the modern chemical and physical laboratory.

Biologists have always associated activity with life, and for many decades now they have

known that a vast amount of action goes on within the living organism. They have been aware of the beating of the heart that keeps the blood in circulation, the rhythmic motion of the lungs that supplies oxygen to the blood stream, the complex chemical activities of the digestive apparatus and other organs. More recently they have learned something about the living drug factories, the ductless glands, and about the electrochemical messages that flow along the nerve fibers.

But new experiments, applying the latest discoveries of atomic physics to the problems of physiology have disclosed a veritable cyclone of activity within the human body such as was never before suspected. The recent discovery of artificial radioactivity has made possible these new findings.

At the Harvard Tercentenary Conference, Professor August Krogh,[9] the distinguished biologist of the University of Copenhagen, reported experiments in which radioactive phosphorus was fed to rats. He reported that within a short time this radioactive phosphorus had left the blood, exchanging places with the ordinary phosphorus of the tissues. This exchange involved the muscles and other organs. Even more astounding, he said, was the fact that this radioactive phosphorus found its ways into the bones and teeth. He believes, therefore, that we must change our views of the structure of living organisms, accepting a constant movement of atoms within it such as was never previously pictured.

It is interesting to speculate what life may be like when our knowledge of the chemistry and physics of the human body has become so great as to give us such control over it as is undreamed of today.

Perhaps the time is coming when it will be possible to make a hormone survey of the growing child. A few drops of his blood, carefully extracted from a pinprick in a finger, placed in a test tube and sent to the laboratory for analysis, may reveal far more about the child than any present-day method. Perhaps by that day, the physician will also possess sufficient knowledge to act upon what the analysis will show.

Who can say how successful these methods may prove eventually? Perhaps the muscles of the strongest child are the rightful heritage of every child, the keenest brain the birthright of every infant.

The whole world is thrilled when a youthful Yehudi Menuhin strides out upon the concert stage, playing the works of Beethoven and Brahms with the brilliant understanding of a mature genius. The whole world stares in amazement when an eight-year-old boy turns out to be a "marvel" at chess, playing fifty simultaneous matches against masters of the game and winning them all.

Perhaps the genius that makes a Yehudi Menuhin or a chess marvel lurks within every child.

If you drive an automobile you have no doubt experienced a time when the car developed engine trouble. The motor sputtered and backfired; it ran haltingly and noisily. You took the car to a garage where a trained mechanic rolled up his sleeves and got out an assortment of tools and wrenches. He regulated the carburetor and made other minor adjustments. Soon the engine began to run smoothly and quietly, quickly gaining its full efficiency.

It may be that every human being is like the automobile engine that is not quite correctly adjusted. It may be that tiny adjustments, if we knew how to make them, would open up the potentiality of genius for every child. Perhaps that hope is extravagant, but there seems every reason to believe that the time is coming when far more will be accomplished to insure stronger bodies, healthier nerves, stabler dispositions, and keener minds for every child than at the present time.

V

Physics derives its fourth great cultural value from the fact that it is the foundation stone of all attempts to understand the universe. We have just been considering the basic relationship which physics bears to chemistry and biology. That relationship applies equally to astronomy and cosmogony. I attempted to trace this essential unity of the universe in my first book, *The Story of Science*.[10]

The universe is one. The same fundamental laws that govern the electrons in the atom, control the stars in the Milky Way. Modern

science has achieved its greatest triumph in tracing the organization of the universe from the tiny electron to the great clouds of galaxies. This has been done with the aid of physics. Perhaps it was in this field that the importance of physics was first most clearly realized. Galileo was an astronomer as well as a physicist and the laws which he and later Newton developed were seen at once to apply to the heavens as well as the earth. Appropriately enough the study of planetary motions was christened "celestial mechanics."

Newton in his law of universal gravitation stated a rule that applies as truly to the double star, five hundred light-years away as it does to the apple falling from the branch of a tree.

The kinship of physics and astronomy became clearer with the investigations of the nature of light and the invention of the spectroscope and this kinship was duly celebrated with the christening of this branch of study as "astrophysics."

In his attempts to solve the problem of the evolution of the galaxy, the genesis of the sun's heat, the origin of the solar system, and many other fascinating problems of the heavens, the astronomer turns to the knowledge which the physicist has accumulated about the behavior of subatomic particles and energy photons. He employs the experimental apparatus of the physics laboratory and the equations of the mathematical physicist.

"Matter and force are the two names of the one Artist who fashions the living as well as the lifeless," wrote the great Huxley. But the modern view puts the greater emphasis upon energy.

"All the life of the universe," says Sir James Jeans, "may be regarded as manifestations of energy masquerading in various forms, and all the changes in the universe as energy running about from one of these forms to the other, but always without altering its total amount."[11]

In our attempts to construct a universe, therefore, we may regard all the various subatomic particles as concentrated energy, "bottled energy" if you please, since the recent experiments with artificial radioactivity have verified Einstein's equation of 1905 for the conversion of matter into energy and vice versa.

It is interesting to ask what ingredients we need for the construction of a universe in addition to energy. A generation ago we should have required space and time, but now we need only the space-time continuum of Einstein.

We need certain forces within this space-time continuum—the force of gravity, electromagnetic forces, the nuclear binding forces which Tuve and his associates have disclosed, and perhaps the cosmic force of repulsion to account for the expanding universe of Lemaitre. For the study of all these, we must turn to physics. And then, perhaps, we shall eventually in the fashion set forth in Einstein's field theory, come to regard all of them as manifestations of space-time field. But whatever decision we may reach eventually, it is apparent that the man without training in physics cannot work successfully in this field and the man without a knowledge of physics cannot hope to have an intelligent understanding of what is being done.

Needless to state, this is a field in which every person, however slight his formal education has been, shows a keen interest. Speaking two years ago before the American Association for the Advancement of Science upon the subject of "Science and the American Press,"[12] I sought to trace the factors which accounted for the present-day widespread interest in science. I pointed out that the interest in Einstein's theory of relativity was one of the chief factors in the rapid growth of interest in science immediately following the World War.

I have tried to show so far that a knowledge of physics is necessary for an understanding of the age in which we live, for an understanding of all science, for an understanding of the future, and for an understanding of the universe in which we live. Before concluding I wish briefly to list certain virtues to be gained from the study of physics. These constitute the remaining cultural values of physics which I want to discuss.

VI

Physics derives its fifth great cultural value from the fact that it teaches the meaning and the value of natural law. This discovery of the existence of the laws of nature has been one of man's greatest triumphs. It has changed his whole intellectual outlook.

To ancient man the universe was a chaos, governed by caprice. In order to explain its phenomena, he found it necessary to people the heavens with a host of minor gods and goddesses, and the mountains and streams with a varied throng of giants, nymphs, and spirits. The occurrence of an eclipse, the appearance of a comet, the gathering of the thunderstorm and the flash of the lightning were interpreted as the activities of these mythological personages.

Gradually science revealed the order of the cosmos. It taught that the universe was orderly, functioning in response to well-established laws. A corollary of the existence of these laws is the important fact that their willful neglect leads automatically to its own inexorable penalty.

A sixth great cultural value of physics arises from the fact that it teaches precision. Experiments must be planned with precision. Observations and measurements must be precise. Thinking must be definite and logical. This is a lesson well worth learning. The student who carries habits of precision from the physical laboratory to the outside world is the richer thereby.

A seventh great cultural value of physics is that it inculcates a love for the truth and a desire to attain it. I have dwelled at some length upon the present and future applications of physics.

To the scientist, the practical applications have always been secondary. He has sought primarily to understand nature and the universe. Galileo, meditating upon the laws of motion, was trying to understand the workings of nature. He was not thinking of engines and machines. Maxwell, seeking to explain the nature of light, had no thought of the radio. This does not mean that science is contemptuous of its practical uses. The opposite is true. But it does mean that the true scientist is motivated by a higher aim than to make life easier. He wishes also to ennoble and to enrich life. The spirit of science then is, first of all, the wish to know, the urge to seek, the desire to comprehend the universe.

I have sometimes noticed that people who have had no training in science, and therefore have no adequate understanding of its spirit, are confused by this point. I have had them come to me, for example, and ask, "What is the practical use of the Einstein theory?" They understand that scientists regard Einstein as the greatest scientific mind since Newton, perhaps as the greatest scientific mind of all time. But they cannot understand that the scientist venerates the great excursions which Einstein has made into the realm of understanding.

The pursuit of physics, therefore, is valuable in that it will inculcate this point of view in the student and give him a richer outlook upon life.

The eighth great cultural value of physics is its ability to instill the spirit of courage in its students. In this respect physics is one with the other sciences, for the scientist has never been bound by ancient tradition. Copernicus dared to cast aside the Ptolemaic theory though it had dominated man's thoughts for centuries. Vesalius challenged the authority of Galen's anatomy even though it had ruled since the time of the Romans. Scientists did not fear Newton's "Principia" because it was new. They did not flee from Maxwell's electromagnetic theory of light because it was revolutionary.

Twentieth century scientists have not rejected Planck and Rutherford and Schrödinger and Einstein because their ideas were new. On the contrary, they have rejoiced in each new discovery. This is the courage which the world needs constantly, the spirit to forge ahead, to discover new truths, and to face them when they have been discovered.

The ninth great cultural value of physics is that it instills the spirit of tolerance. The physicist knows that there is no monopoly upon truth. He sees the advance of science as a great cooperative venture of all nations and peoples down through the years. The roll of every science is an international one. Copernicus was a Pole, Tycho, a Dane, Kepler, a German, Galileo an Italian, Newton, an Englishman. The story is the same today. The theory of Einstein receives its chief verifications at the hands of English and American scientists.

The scientist is tolerant of other men's points of view. Realizing how frequently he must change his own views in the face of new evidence, he is never scornful of the other man's point of view. He realizes how little mankind knows and how much is yet to be learned and the realization makes him tolerant.

The twentieth century physicist is peculiarly aware of the danger of jumping to dogmatic and sweeping conclusions on insufficient evidence. He is cognizant of the mistake which the nineteenth century physicist made in concluding that the structure of physics had been completed and that he was justified from that structure in believing in a purely mechanistic universe.

Today, the physicist is aware of the change in our thinking which has been introduced by the Einstein theory of relativity and the Heisenberg uncertainty principle.

Proud as he is of the precision of his experiments and his thinking he realizes that there is, seemingly today, a place where precision breaks down. He knows, from the Heisenberg principle of uncertainty, that he can never measure both the position and the velocity of an electron with exactness. What he achieves in exactness in measuring position, he loses in exactness in velocity, and vice versa.

He is careful not to jump to conclusions too quickly from this fact, although physicists everywhere are studying it. Thus Bohr, for example, has extended this principle to other pairs of measurements and calls these paired quantities "conjugate quantities," and the relationship between the two "complementary."

What may come of this we do not yet know. It is a strange fact indeed that Planck's constant enters the picture at this point. The product of the uncertainty in the case of two conjugate quantities is never less than Planck's constant. It appears to set a natural limit on the exactness of measurement in the atomic world.

The physicist is impressed by many other problems awaiting solutions and for these reasons, therefore, his spirit is the spirit of tolerance.

And finally we come to the tenth great cultural value of physics. This value arises from the fact that the spirit of physics is the spirit of humanity.

Einstein taught us that the observer is always part of the experiment. There is no such thing as setting up an experiment which is a closed system independent of the observer. While the physicist may have thought that possible in the past, nevertheless there never was a time when the physicist forgot human values.

The physicist has always been concerned for the future of mankind. The picture of the scientist as a man who shuts himself away like a hermit in a cave is an unfair picture. There are, of course, such individuals but they are not representative of science.

Let Einstein, whose theories represent man's greatest flight today into the world of the abstract, speak for the scientist's interest in the concrete facts of life. In February, 1931, while visiting in Pasadena, he addressed the students of the California Institute of Technology.

"Why does this magnificent applied science which saves work, and makes life easier, bring us so little happiness?" he said. "The simple answer is: Because we have not yet learned to make sensible use of it."

"It is not enough that you should understand about applied science, in order that your work may increase man's blessings," Einstein told the students. "Concern for the man himself and his fate must always form the chief interest of all technical endeavors. Never forget this in the midst of your diagrams and equations."

How much the world needs this spirit today is evident if we turn our attention to recent events in Europe. One cannot resist comparing these words of Einstein's with the words of the man who drove him out of Germany. No doubt you heard his speech of hate, filled with the rattle of the saber and the threat of war, that was broadcast to the world during the Czechoslovak crisis.

A decade ago, H. G. Wells pictured the world in a race between education and destruction. That afternoon, as those words of hate boomed forth from radios everywhere, it seemed as though destruction was about to win.

The dictators of Europe have made no secret of their contempt for democracy and for the freedom of thought and expression which is not only the life of democracy but the life of all science as well.

But will destruction win in the end? I am one who does not think so. I am certain that it will not so long as America remains faithful to its belief in democracy. Therein lies the importance of our educational system for what we teach the young men and women in the schools today will determine the conviction of the citizens of tomorrow.

VII

I have urged in this address that we shape our educational system so that physics be given the position once occupied by the classics as the common cultural bond that united all educated men. In conclusion let me list the ten cultural values which I have discussed and which, in my opinion, justify this place of honor for the science of physics: (1) Physics is the foundation of the present age and a knowledge of physics is necessary for its complete understanding. (2) Physics seems to be the foundation of every science and all of them can be better understood with an understanding of the principles of physics. (3) The greatest advances of the future will probably be based upon the new discoveries of physics. (4) Our understanding of the cosmos and the picture of the universe given by modern cosmology is founded upon the science of physics. (5) Physics teaches the importance of natural law. (6) Physics teaches precision in observation, experimentation, and deduction. (7) The spirit of physics is the search for the truth. (8) The spirit of physics is the spirit of courage. (9) The spirit of physics is the spirit of tolerance. And finally, (10) The spirit of physics is the spirit of humanity.

I call upon you to have courage and to labor with faith for the future of civilization, for the dawn of that day when the spirit of physics, the spirit of all science, will triumph over the forces of blind hatred, of cruel violence, of bigotry and intolerance. God grant that the dawn may be soon.

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An Experimental Electronic Violin

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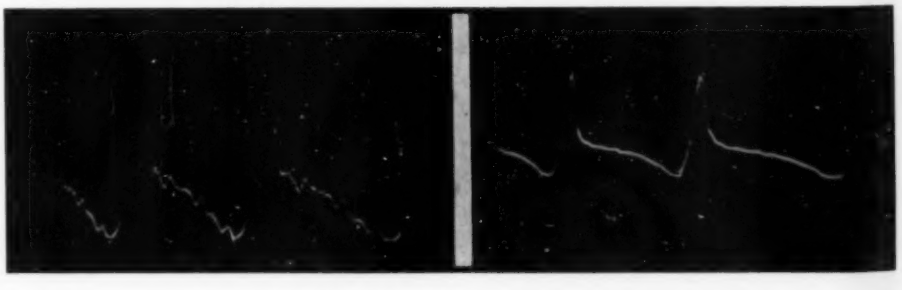
MODERN electroacoustical techniques have accomplished wonders in the transmission, recording, and reproduction of sound but have contributed less than might be expected to the design and performance of conventional musical instruments. This is especially true of the violin family which reached its highest state of perfection late in the seventeenth century, in the hands of the great Italian makers. The intervening years have seen minor improvements but there have been no really significant changes in design or construction.

Studies, both experimental and theoretical, of the acoustical characteristics of the violin, viola, and cello have been made by engineers and physicists. Wave forms have been recorded and analyzed and valuable old fiddles have been dissected, in attempts to explain the variations in tonal quality found among different instruments. No notable improvements seem to have resulted, however, and modern violin making still consists, mainly, of copying the form and construction of the old master violins. Unfortunately, these modern replicas seldom approach the excellence that might be expected of them; the variations in tonal characteristics from one instrument to another being difficult to control or predict. Even the most highly prized Cremona violin may leave something to be desired in the way of volume range and tonal variety.

Frequency Response Defects

The vibratile system of the violin family consists of four stretched strings coupled by a light wooden bridge to a pair of thin wooden plates (belly and back) with an air cavity between. Back and belly are connected by a mechanical linkage, the sound post. The front or belly is re-enforced inside by a light wooden beam called the "bass bar" which runs longitudinally about two-thirds the length of the body. Such a system might well be expected to exhibit unpredictable irregularities in frequency response.

Under the action of the bow the strings execute a type of motion akin to relaxation oscillations. The bow catches the string and draws it aside until its restoring force exceeds the force of static friction. It then breaks away, swings back beyond the rest position, starts to return and is caught by the bow again. Since the bow is moving with nearly constant velocity, and the string moves with it during a portion of the cycle, it is clear that part of the displacement-time curve for the string will be a straight line. Fig. 1 shows oscillographic records of the motion of a point near the bridge on a standard violin D-string, taken under normal playing conditions. The other strings yield wave forms having the same general characteristics. Trace (*a*) was made with a high bow velocity while trace (*b*) was made with a lower velocity. The slope of the straight



(*a*) (*b*)

FIG. 1. Oscillograms showing motion of a point on a violin D-string. (*a*) with high bow velocity, (*b*) with lower bow velocity.

portion of each curve is, of course, proportional to the bow speed. An electrostatic pick-up, to be described, was used to obtain these records. It is evident that a harmonic analysis of the curves shown in Fig. 1 would show a relatively strong fundamental with diminishing amplitudes for the higher harmonics.

The wave forms shown in Fig. 1 are quite typical of the motions executed by all of the strings, under a wide variety of bowing conditions. This cannot be said of sound wave forms radiated from the body of the violin. Not only do these wave forms vary from one string to another but from one frequency to another on the same string.

To understand the transitions that take place between the vibrating string and the radiated sound wave, the string-bridge-soundboard system may be regarded as a transmission line coupled to a multi-resonant network. The mechanical complexities of violin construction, as well as the inhomogeneities in materials, would make it difficult to represent all of the elements of such a network, but the analog is useful, if only to show the difficulties involved in any kind of quantitative analysis. It is interesting to note, however, the possibility of determining, experimentally, the transmission characteristics of a violin for different simple harmonic driving frequencies applied to the bridge, just as we now determine the characteristics of a loudspeaker. This would provide a basis for the comparison of different instruments.

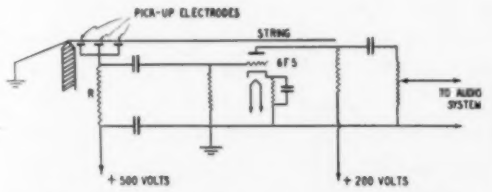


FIG. 2. Circuit schematic for electrostatic string pick-up.

Judged by the criteria used to select a good cone type loudspeaker the frequency-response characteristic of the average fiddle would be rated as very unsatisfactory. Experimental studies have demonstrated[1] quite conclusively that the excruciating tones of a bad fiddle usually result from strong resonance humps and poor low frequency response. The better instruments are characterized by a somewhat more uniform response over the entire frequency spectrum above 196 cycles.

FIG. 3. An experimental model.

If one should go so far as to postulate that the ideal violin, viola, or cello is one having, for example, a flat frequency-response curve, then electronics provides a solution to the problem of how to make a better violin. The electroacoustical designer would use the same method of attack that was used to transform the phonograph reproducer from the old diaphragm type to the modern electrical reproducer. The advantages of dealing with vibrations in the electrical form are evident. Once in this plastic form they can be molded by an electrical network having transmission characteristics that can be accurately controlled. Indeed, all the experience from years of communication engineering may be used, to transform the violin from its time-honored form into a modern musical instrument.

[1] P. Jarnak, "The Violin," J. Frank. Inst. **225**, 315–342 (March, 1938). Also, P. H. Edwards, Phys. Rev. **32**, 23–37 (1911). C. W. Hewlett, Phys. Rev. **35**, 359–372 (1912).

Design Considerations

Obviously, the application of an electro-acoustical transducer to some point on the back or belly of the instrument offers little advantage beyond a mere increase in volume. This slight advantage is likely to be offset by serious feed-back difficulties. Since the motion of the bridge is affected by the reaction of the structure which supports it, electrical translation at this point has very limited possibilities. The mechanical impedance of the bridge support may be increased by the use of a rigid framework in place of the soundbox. This reduces the effects of body reaction but difficulties arise due to the variable impedance of the bridge itself. Pronounced resonance effects are usually present unless artificial damping is resorted to.

Direct inductive pick-up from the vibrating strings offers several important advantages. The effects of variable bridge and body reaction are largely eliminated, if a bridge having high mechanical impedance is used; the output of each individual string may be independently controlled; and the pick-ups have no appreciable effect on the motion of the strings.

Either electromagnetic or electrostatic inductive effects may be employed to translate the string vibrations. The latter effect affords the simplest, most compact and flexible design for small stringed instruments. B. F. Miessner, who has used electrostatic pick-up methods on his electronic piano[2] as well as on violins,[3] has pointed out the advantages of direct electrostatic translation of the string vibrations.

A simple form of electrostatic pick-up consists of a small metal probe or electrode placed on an insulating support near the vibrating string. This pick-up electrode is charged to a potential of several hundred volts through a high resistance R, as shown in Fig. 2. The principle of operation is similar to that of the condenser microphone. As the string vibrates, the varying capacitance between it and the pick-up electrode induces a small alternating potential difference across R. This voltage is applied through blocking condensers to the grid of an amplifying tube. As in the case of the condenser microphone, the pre-amplifier tube must be located close to the charged electrode because of the small capacitance between it and the string.

With these principles in mind an experimental violin, shown in Fig. 3, has been constructed. Standard violin parts were used, with the exception of the body and bridge. The heavy fiber bridge, of special design, (Fig. 4) is separated from the solid wooden chassis by a strip of felt.

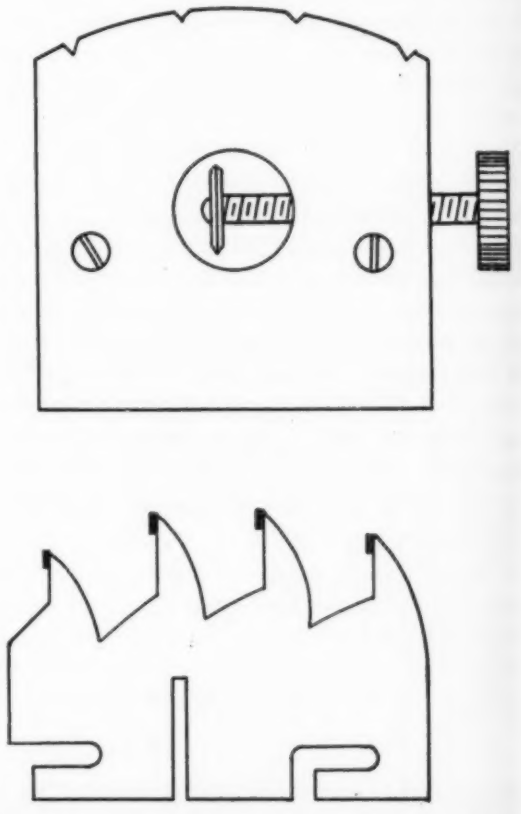


FIG. 4. Diagram of special bridge showing spacing adjustment and demountable pick-up plate.

Holes are drilled through the chassis to receive a 6F5 tube and its associated resistors. Two condensers are also mounted on the instrument. A four-conductor shielded cable carries filament current and plate voltage for the 6F5 as well as a 500-volt charging potential for the pick-ups. A

[2] B. F. Miessner, "The Application of Electronics to the Piano," Proc. Rad. Club Am. **11**, No. 1 (1934). Also, "The Design Considerations for a Simple and Versatile Electronic Music Instrument," J. Acous. Soc. Am. **6**, 181–188 (1935).

[3] Mr. Miessner has experimented extensively with electronic violins employing direct electrostatic pick-up from the strings. He holds a number of important patents in this field, and has recently informed the author that his licensees are undertaking commercial production of one of his electronic violins.

small power pack supplies these voltages. Aluminum wound gut strings are used, with the exception of the E string, which is of steel. The strings are connected at the tailpiece to the grounded cable shield.

Since the steady-state motion of a vibrating string may be regarded as the resultant of a number of standing wave systems, it is clear that a given harmonic will be translated with greatest efficiency when the pick-up is at an antinode for the standing wave that would generate that particular frequency. All harmonics have nodes at the ends of the string, and all even numbered ones have a node at the midpoint. In general the nth partial has $n+1$ nodes (including end points) equally spaced along the string, with n antinodes between. Those harmonics having nodes near the pick-up point will be suppressed while those having antinodes at, or near, this point will be translated according to their relative amplitudes. The structural relations of strings and fingerboard assembly limit the position of pick-up points to the lower $2\frac{1}{2}$ inches of string between the end of the fingerboard and the bridge.

In devising pick-up structures for an experimental instrument, it is desirable to provide for both transverse and longitudinal adjustment of the electrodes so that the string-to-electrode spacing, as well as the pick-up point, may be varied. Some of the earlier designs included three electrodes for each string with individual adjustment for each electrode. Experiments with these demonstrated that a variety of tone colors could be obtained by varying the number, and relative positions, of the pick-up electrodes. A single narrow electrode placed near the bridge yields a tone which may be characterized as "thin" or "sharp" in quality while the same shape of electrode located at the end of the fingerboard yields a soft mellow tone. These two tone qualities may be combined by connecting both pick-ups together.

Objections raised by violinists to the multiplicity of adjustments on the earlier models, with individual pick-up adjustments for each string, led to the present design which provides for the simultaneous use of from one to three composite pick-ups. In this type of pick-up the electrodes (one for each string) take the form of teeth projecting from the edge of a brass plate. In some cases the teeth are built up to a thickness exceeding that of the plate ($\frac{1}{8}$ to $\frac{1}{4}$ inch) to increase the string-to-electrode capacitance. These composite pick-up plates are mounted on insulating supports in such a way that the teeth fall in a line perpendicular to the strings with a tooth

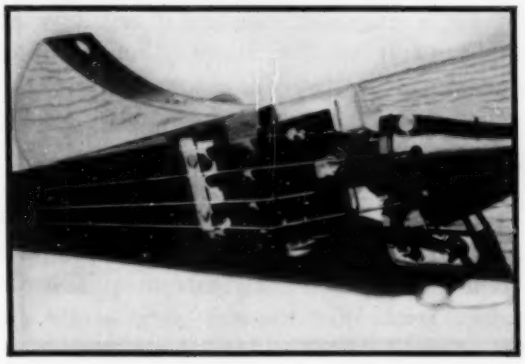


FIG. 5. Close-up showing special bridge and pick-up structures. Only two of the three pick-up plates are in place.

adjacent to each string. Slots in the plates provide for both transverse and longitudinal adjustment with respect to the strings. The present model has three of these pick-ups, one mounted on the bridge and two between bridge and fingerboard.

Because of its proximity to the ends of the strings the bridge pick-up requires close string-to-electrode spacing; it is, therefore, provided with screw motion. Fig. 4 shows the details of its construction. A thumbscrew, threaded into the bridge, carries at its end a small fiber disk which engages a slot in the pick-up plate. The plate is held to the face of the bridge by a bayonet-type fastening which permits transverse motion. Any one of a set of interchangeable pick-up plates, having various shapes and electrode sizes, can be quickly slipped into place or removed with the fingers.

The other two pick-ups are of similar design except that they are not provided with screw motion adjustment. Each has a right angle bend and is mounted between two brass strips on top of a Bakelite standard. The photograph (Fig. 5) shows one of these in place. These last mentioned pick-ups are used to re-enforce the fundamental and lower harmonics and do not require frequent or critical adjustment. Provision is made for

shifting the Bakelite support as well as the individual pick-up plates.

Each pick-up plate requires careful adjustment before it is put into use. The width and spacing of the electrodes must be so proportioned that a proper balance exists between the tones from the several strings; the electrodes must not strike the vibrating strings, nor interfere in any way with bowing operations. Voicing or balancing adjustments are made by ear; electrodes giving too much pick-up being filed down until a proper balance is obtained.

Performance

When played through a high quality audio system the experimental instrument described produces violin-like tones of surprisingly good musical quality. With all three pick-ups in operation the tones are rich and full bodied, due to efficient translation of all components of the strings' motion. Removal of the bridge pick-up reduces the output of higher harmonics, yielding a soft muted tone. If the bridge pick-up, alone, is used the tone quality is more brilliant and piercing due to greater translation efficiency for the upper partials when the pick-up point is near the end of the strings. A variety of intermediate effects can be obtained by varying the string-to-electrode spacings of the three pick-ups. In addition, single pick-up plates with different electrode arrangements produce characteristic tonal effects depending upon their positions with respect to the strings.

All the effects producible on an ordinary violin are possible with the new instrument; also, it is easier to play, requiring less finger pressure and less bowing effort. Violinists who have played it are captivated by its tonal possibilities, volume, and ease of playing. When played as a viola its tonal effects are especially pleasing because of the efficiency with which the pick-ups translate the lower frequencies.

The expressiveness and general performance of the instrument can doubtless be improved by the use of a volume expander in the amplifier circuit. This would help to bring the full dynamic range of the amplifiers under control of the performer's bow. Plans are now under way to include this feature in the present set-up.

In addition to revealing interesting possibilities as a new and versatile member of the violin family, this experimental instrument has cast some light upon the peculiarities and weaknesses of the conventional violin. If a suitable filter is inserted in the output circuit of the electronic violin, so that the lower frequencies may be suppressed to any desired degree, the lower tones obtainable on the G- and D-strings have a quality very similar to the corresponding tones from a standard violin. This confirms the findings of other investigators,[1] that the vibratile system of the conventional violin translates the lower frequencies with rather poor efficiency.

Violinists and conductors who have tried the new electronic instrument in a large auditorium report that it has promising possibilities for orchestral, as well as solo use. One of the outstanding limitations of the ordinary violin is its low acoustic output. This limitation is keenly felt when a violin concerto is played with orchestral accompaniment or when a violinist performs in a large auditorium or out-of-doors. Electronic instruments are, of course, not subject to such limitations but suffer such restrictions as limited portability, slightly greater weight, and interference from electrical disturbances. It seems quite possible that with the benefits of a distinctive and artistic body design, making use of plastic construction and attractively finished metal parts, the electronic members of the violin family may assume an important place among modern musical instruments.

Resumés of Recent Research

Recent advances in experimental and theoretical physics are described in these columns in nontechnical language. It is intended that sufficient background be given to introduce the reader to the subject.

Recent Experiments in Artificial Daylighting

Color is an important quality factor in the grading of agricultural products, and favorable conditions of illumination are therefore necessary for grading these products. Satisfactory natural illumination is so important that some industries go to great lengths to provide it. Many commercial cotton classing rooms, for example, are supplied with specially constructed "government-type skylights." They are long, set facing due north, at a definite angle from the horizontal, with a circular back to diffuse the light, the rooms being painted a neutral gray so that no one chromatic color may be discounted or flattered more than another.

Although such rooms have proved valuable, they are not possible for all situations, and for many years the use of artificial daylighting has been considered. Since data on which to base specifications were lacking, a study has been made recently by Dr. Dorothy Nickerson in the United States Department of Agriculture in order to supply this lack, and although the report of the work[1] is confined chiefly to work with cotton, it is expected that the solution of the cotton problem, with only minor adjustments, would provide an answer for other agricultural commodities. This will be true if relative energy values of the artificial illumination approximate those of natural daylight. If they do not, a satisfactory illuminant for one product may not prove satisfactory for another.

Certain specifications were already available, but information regarding intensity was lacking. Measurements of illumination intensities were therefore made daily in many well-lighted classing rooms from New York to Texas. The data were accompanied always by a record of sky and classing conditions in order that the figures could be broken down to give desirable and undesirable intensities for classing conditions on clear, cloudy, and overcast days, etc. It soon became evident that something more than *amount* of light makes conditions good or poor. Intensities reported in June were generally twice as high as for the same conditions in December: a "very good" classing day with clear weather in June might measure between 108–150 footcandles horizontal illumination on the tables, but in December it required only 41–50 footcandles to be called "very good." The same ratio held good for slightly cloudy, and for overcast days.

The specifications finally proposed include: (1) diffuse illumination nearly uniform over a wide area; (2) color to match slightly overcast north sky; (3) intensity of 60–80 footcandles at a horizontal working plane with little advantage in intensities increased above 100 footcandles.

After consideration, a choice of lamp-and-filter combination was made to meet these

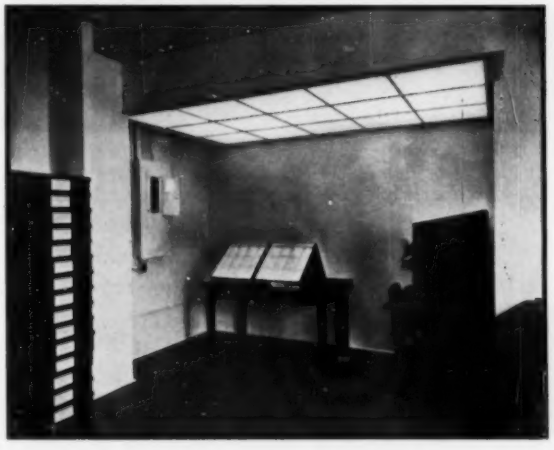


FIG. 1. Artificial daylight installation in Washington aboratories, Division of Cotton Marketing, U. S. Department of Agriculture. This type of installation was developed for use in the color grading of agricultural products.

[1] Dorothy Nickerson, J. Opt. Soc. Am. **29**, 1 (1939).

specifications. By trial of varying filter thicknesses it was possible to arrive at a satisfactory match for overcast sky. The installation shown in Figs. 1 and 2 (in which the outer rows are 1000-watt lamps, the inner series 500-watt lamps) give a footcandle distribution 32 inches from the floor (diffusing glass set 8 feet from the floor) of approximately 60 footcandles, diffuse illumination over an area 6×11 feet, a color temperature of 7400°K, with an energy curve approaching that of natural daylight, not far different from I.C.I. Illuminant "C" though somewhat higher in color temperature. A first installation, nearer to Illuminant "C" was rejected, by actual trial, because it was too low in color temperature.

A study of color temperatures and the energy distributions of several sources of illumination other than lamp-and-filter combination was made and is reported as part of the original report. References to much of the most recent work are made. Curves are included for standards generally referred to in work with daylight, to Macbeth

FIG. 2. Looking into the lamp-and-filter arrangement, above the diffusing glass "skylight" shown in Fig. 1.

and Trutint filters, to Sunshine carbon arc, Carbon Dioxide, as well as the more recent fluorescent "Daylight" tubes.

The original paper suggests that a standard be established for artificial daylight that will provide a reference point for use in meeting the needs of grading and inspection work in fields in which color is an important grade or quality factor. Such a standard might provide a "D" Illuminant, added to the "A," "B" and "C" already adopted by the International Commission on Illumination as reference points, "A" representing light from a gas-filled incandescent lamp, "B" noon sunlight, and "C" average daylight. "D" would represent overcast north sky.

Subtractive Color Mixture and Color Reproduction

Among his many significant contributions to physics, J. C. Maxwell included a series of researches on color vision, and as a result invented the basic principle of three-color photography. Maxwell suggested projecting together on a screen three pictures of the same scene through three lantern-slide projectors equipped with red, green and blue light filters. This "additive" process was practiced successfully by many photographers, and several modifications based on Maxwell's suggestions are in use today. Many photographers felt that the same result could be accomplished by printing together on a single paper or film support three photographic images formed of colored materials, each of which would absorb light in only one of the regions of the spectrum specified by Maxwell. Many of the most important modern processes of color photography utilize this "subtractive" principle.

Unfortunately, the absorptions of the colored materials used in forming the partial images are not confined to the most appropriate portions of the spectrum. Thus, the yellow dyes which are used to absorb blue light in color photographs have more or less serious absorptions in the green region of the spectrum. The magenta, or pink dyes which are used to absorb green light also absorb both blue and red light more or less seriously. Finally, the cyan, or bluish green dyes which are used to absorb red light also absorb both blue and green light. These undesirable absorptions of all of the dyes and pigments available for color photography not only render the theoretical discussion of the subtractive process very difficult, but they also result in serious limitations and errors of color reproduction.

A recent investigation[1] by Dr. David L. Mac Adam of the Eastman Kodak Company of the

[1] David L. MacAdam, J. Opt. Soc. Am. **28**, 466 (1938).

manner in which colors are produced by combinations of typical dyes used in subtractive color photography resulted in the formulation of an approximate empirical rule by which the color resulting from such a mixture could be expressed mathematically in terms of the concentrations of the three dyes. An application of this rule to the theory of color photography led to the conclusion that compensations for the absorptions of the dyes in the undesired portions of the spectrum could be accomplished by use of a modification of a photographic technique which has been known and practiced to some extent for over twenty-five years. This method of compensation is known as masking and consists of the use of low contrast positive transparent images bound in contact with one or more of the photographic negatives used for the production of the colored images. The contrasts and other characteristics of the compensating masks used with each negative can be computed from formulas which were derived from the new empirical rule for three-color subtractive mixtures. The density of each point in the mask superimposed on the negative made with the green filter decreases the concentration of the deposited magenta dye to an extent determined by the green absorptions of the yellow and cyan dyes also present at the corresponding point in the color photograph. The mask on each of the other negatives serves the same purpose, to decrease suitably the concentrations of the corresponding dye in all parts of the picture where undesirable absorptions of the other two dyes occur. Since a mask is applied to each negative, the compensations which must be based on the corrected concentrations of each dye involve many interdependent factors, but the results of the investigation are easily applied to practical color photography. The use of the masking method which is recommended as a result of this study will probably be confined to motionless color pictures, such as magazine illustrations and studio prints.

Lithium Fluoride-Quartz Achromatic Lenses

In optical work within the visible and air-transmitted ultraviolet regions of the spectrum it is common practice to employ quartz lenses. Focusing is sometimes a bothersome problem which often has to be solved by time consuming cut-and-try methods. Although fluorite can be combined with quartz to form an achromatic lens, the former is becoming so scarce that there is little hope at present that such lenses will become cheap enough to permit widespread use. Furthermore, the diameter of a fluorite-quartz achromat is definitely limited because fluorite crystals of large size and suitable optical quality are rare.

That lithium fluoride, an artificial crystalline material which is now available commerically, can be substituted for fluorite in making achromatic lenses has been pointed out in a note* published by Professor D. C. Stockbarger and Dr. C. H. Cartwright. This opens the way to relatively inexpensive, well-corrected optical systems for ultraviolet work. Furthermore, since large lithium fluoride lens blanks can be had, the diameters of the achromatic combinations should be limited only by the sizes of the quartz crystals.

Present indications are that lithium fluoride can be combined with fluorite for work within the Schumann region, down to 1300A. There having been no achromatic lenses which could be used in this part of the spectrum, this contribution may lead to interesting new developments.

* D. C. Stockbarger and C. H. Cartwright, J. Opt. Soc. Am. **29**, 28 (1939).

The great adventure is not so much the joy of understanding as the excitement of endless search; and correspondingly, to science the wonder of knowledge is not the richness of its present revelation but the limitless possibilities of its self-fertilizing growth.

—FREDERICK BARRY
in *Scientific Habit of Thought*

New Books

Moderne Mehrgitter—Elektronenroehren II (Modern Multiple Electron Tubes, Vol. 2). DR. M. J. O. STRUTT. Pp. 143, Figs. 98. Julius Springer, Berlin, Germany. Price RM 13.50.

The second volume of *Moderne Mehrgitter-Elektronenroehren* by Dr. Strutt, member of the research staff of the Phillips Incandescent Lamp Works, Ltd., of Eindhoven, Holland, lives up to the quality of the first volume.

The first volume was concerned principally with the static characteristics of multiple grid tubes and their external behavior at low radiofrequencies. The purpose of the second volume as expressed in the author's foreword is twofold: to show the relation between the static characteristics of a tube and its internal construction, and to examine the effect of the rather complex behavior of electrons at high radiofrequencies upon the operation of the tube at these short wave-lengths.

The first 7 articles of the book contain a compact summary of design considerations for tubes to be used at the low radiofrequencies. These contain sections dealing with fundamental electron movements, electrostatic fields, space charge effects, and the various inter-electrode reactions of multiple grid tubes. The sections on this last subject are particularly good.

The second function of the book is fulfilled by the remaining 11 articles which comprise some two-thirds of the book. Particular attention is paid to the subject of tube dynamic capacities for wave-lengths of about 10 meters. A masterful analysis is given of the effect of the lead inductances and capacities upon the external electrode admittances. Powerful use is made of "active admittances" which are defined as the difference between the hot and cold admittances of the tube. A refreshingly simple and clear-cut treatment is given of the effect of alternating fields upon the movement of electrons with emphasis upon the resultant energy losses and gains of these electrons. Good experimental checks, mostly Dr. Strutt's own work, are given for these analyses.

The correctness of the above treatment is evidenced by the characteristics of some new Phillips tubes which resulted from the above studies and experiments. These tubes differ greatly in form from the conventional multiple grid tubes and are not subject to the inherent limitations of the latter. The new tubes have the same magnitude of transconductance at 10 meters that they have at the low radiofrequencies.

Closing articles deal briefly with some remarks on shot effect, and electrode temperatures at high frequencies. Mention is made of a super-secondary emission obtained with an electrical double layer at the secondary emitting surface.

Like the first volume the book is compactly written and well illustrated. There are frequent large gaps in the author's developments. Both the text and illustrations are necessary for a complete understanding of the material. References, while given, are not individually indicated. The author tacitly assumes a complete theoretical background on the part of the reader as well as a familiarity with the details of the many recent developments in the field. Greatest attention is naturally given to matters which the author has personally investigated.

The high frequencies referred to in the book are the borderline frequencies in the vicinity of 30 megacycles between the low and ultra-high radiofrequencies. The book contains much material not readily available in this country. It should be valuable to anyone interested in the effects in multiple grid tubes operating at high radiofrequencies.

KARL SPANGENBERG
Electrical Engineering Department, Stanford University

On Understanding Physics. W. H. WATSON. Pp. 141 +xii. The University Press, Cambridge and the Macmillan Company, New York, 1938. Price $2.75.

After reading this book it is difficult to escape the feeling that it is only the combined qualifications of a philosopher, a logician, a psychologist, a mathematician and a physicist which might suffice in making a proper analysis of it. Even in that case the result might prove highly controversial. The author deserves to be commended for his venture, and all physicists should subject themselves to it. It is to this group the author makes his appeal. As far as the general public is concerned, it is clear that the book is not to prove a best seller. Nevertheless, it would be very desirable if the general public could get to understand more adequately the essential strategy involved in revealing and presenting the so-called facts in nature and the status of these as regards finality. That the physicist should be aware of what he is doing or attempting to do is, of course, obvious. The fact that he is not always so aware makes this volume timely.

Estimates of how well the author has succeeded in the task will, of course, differ. To the uninitiated it will no doubt seem rather involved. These, at the end of the reading of it, will probably feel like the old lady after a sermon when asked what it was all about. Her reply was, "You see when you pour water into a sieve the water goes

through, but the sieve is cleaner." The writer must confess an enhancement of his own cleanliness.

The table of contents reads: I Discipline in Philosophy, II Logic and Psychology and Physics, III On Methods of Representation, IV The Nature of Mechanism, V The Logic of Substance and Motion, and VI Some Aspects of the Symbolism of Mechanics and Electricity.

In discussing topics like these it is, of course, vital that an imperfect and complicated tool like language be used with the highest possible precision. The author will suffer somewhat on this score perhaps mainly because of the reader's own limitations.

The writer feels that the issuing of this volume is abundantly worth while.

HENRY A. ERIKSON
University of Minnesota

An Introduction to Industrial Rheology. G. W. SCOTT BLAIR. Foreword by E. C. Bingham. Pp. 143+xiii. Figs. 20, 12×18½ cm. P. Blakiston & Sons, Philadelphia, 1938. Price $2.25.

Ten lectures on rheology given recently by the author at the Sir John Cass Technical Institute, London, comprise most of this elementary treatise. While written ostensibly for the "practical man" as an introduction to rheological methods and problems, it can be described as a digest of numerous experimental facts and theoretical speculations, ranging from "plasticity of hot coal" to "psychological perception of stickiness." The lay reader will discover many interesting aspects of flow and deformation phenomena that are typical of industrial processes, particularly as regards the rheological properties of food materials, Scott Blair's field of research. The industrial rheologist will find little of practical importance; there are no tables of data and conversion factors, and little criticism of experimental techniques and methods. The A.S.T.M. specifications or governmental standards for rheological materials, for example, would be useful in a book of this kind.

From a critical standpoint, the book does not set a high standard. There has been little attempt toward systematic classification and clarity of thought. Two examples of careless errors: p. 114 "If two atoms approach each other, the resultant of the attractive and repulsive forces takes the form of a rise in the attractive force, until a certain optimum distance is reached. As the atoms approach still closer, the attractive force falls and reaches zero, after which a rapidly increasing repulsive force is experienced;" on p. 15, "For all liquids except water the viscosity increases with pressure." Superficial statements such as, "Oiliness must be related to viscosity," and "All kinds of shapes are found in the curves for viscosities of liquid mixtures: steady rise, fall, maxima, minima, or more complex curves," are not conducive to an appreciation of the author's efforts. After reading the book, the reviewer realizes the relevancy of Professor Bingham's remarks in the Foreword, "good clear-cut definitions—a certain minimum nomenclature to avoid either confusion or circumlocution" are necessary if rheology deserves to become a science.

R. B. DOW
The Pennsylvania State College

Gaseous Electrical Conductors. E. L. E. WHEATCROFT. Pp. 265+xi, Figs. 140, 16×24 cm. Oxford at the Clarendon Press, 1938. Price $6.50.

The metallic conductors which the electrical power engineer commonly uses for carrying his current have electrical properties sufficiently simple and sufficiently unvarying so that there is no pressing need for the engineer to delve into the physical theory of the electrical conductivity of metals. Ohm's law and a few numerical specific resistivities and temperature coefficients are quite sufficient for the designer of motor, generator, or transformer.

When, however, gaseous electrical conductors enter his apparatus, as in circuit breakers, rectifiers, or as the lightning flash which paralyzes his transmission system, the engineer must drop his aloofness, and appeal loudly to the present-day physicist for help, or do some physics research himself. The complexities of conduction in gases are so great, that even only for the purpose of keeping the experimental facts well in mind, some physical theory, dealing with atoms, ions and electrons, is necessary.

The present volume aims to supply the electrical engineer with such physical theory as is now available, and which will be helpful, or which promises to be helpful in the design of engineering apparatus involving gaseous conductors. As the author states, however, much engineering practice has run ahead of theory, so that much of the theory must be classified as "promising to be helpful" rather than actually "helpful."

The book is divided into two parts; the first dealing with physical theory, and the second discussing engineering apparatus with gaseous conductors. All important parts of the theory seem to be included in 149 pages, so that obviously space required that the treatment of each topic be short. The engineering apparatus described in the second part includes vacuum and gas-filled tubes, mercury arc rectifiers, circuit breakers, and luminous discharge tubes. Because of their practical importance here, an American engineer would like to have seen a discussion of lightning and lightning protective devices also included.

JOSEPH SLEPIAN
Westinghouse Electric & Manufacturing Company

Among Our Authors

David Dietz has been science editor of the Scripps-Howard Newspapers since 1921. He was born in Cleveland, Ohio, and received his B.A. degree from Western Reserve University. He is the author of two books, *The Story of Science*, and *Medical Magic*. He was co-winner of the Pulitzer Prize in Journalism in 1937.

Lynn H. Stauffer received his B.S. degree from the Utah State Agricultural College in 1927 and his Ph.D. from the University of California in 1930. He is, at present, Assistant Professor of Physics at the University of Idaho.

Sturt T. Martin, Jr., was born August 4, 1913, in Brooklyn, New York. He received the degrees of B.S. and Sc.D. in Physics from the Massachusetts Institute of Technology in 1934 and 1938, respectively. He was a Charles A. Coffin Research Fellow in 1937–1938. At present he is an instructor of physics at the Clark University.

C. E. Fink

L. B. Headrick

Louis B. Slichter received his B.A., M.A. and Ph.D. degrees from the University of Wisconsin in 1917, 1920 and 1922, respectively. He served at the U. S. Naval Experiment Station from 1917 to 1919, and at the laboratory of the Submarine Signal Corporation, Boston, 1922 to 1924. He engaged in geophysical prospecting work as a member of the firm of Mason, Slichter and Gauld from 1924 to 1931; was Associate Professor of Geophysics at Massachusetts Institute of Technology from 1931 to 1932, and Professor of Geophysics from 1932 to date.

Lewis B. Headrick received his B.S. degree from the University of Chattanooga in 1926, his M.S. and Ph.D. degrees from the University of Michigan in 1928 and 1930, respectively. From 1930 to 1931 he was employed in the department of engineering manufacture of Western Electric Company. Since 1931 he has been with the Research Engineering Department of the RCA Manufacturing Company at Harrison, New Jersey.

C. E. Fink received his B.S. and M.S. in Chemical Engineering from the Pennsylvania State College in 1932 and from the Massachusetts Institute of Technology in 1933, respectively. For two years he served in the capacity of production engineer in petroleum refining for Atlantic Refining Company in Philadelphia, Pennsylvania and for the past three years he has been working on research in lubricating oils at the Petroleum Refining Laboratory of the Pennsylvania State College.

H. W. Farwell is Professor of Physics at Columbia University, where he has served in various grades since 1906. His chief activity has been with undergraduate teaching. Published papers in the field of optics indicate his general interest.

L. H. Stauffer

L. B. Slichter

S. T. Martin, Jr.

D. Dietz

Simon Ramo was born in Salt Lake City, Utah, in 1913. He received a B.S. degree at the University of Utah and a Ph.D. at California Institute of Technology where he held a teaching fellowship. Since graduation he has been employed by the General Electric Company as a supervisor in their Advanced Course in Engineering and in high frequency research.

A. M. Weinberg

C. L. Pekeris

S. Ramo

Chaim L. Pekeris received his B.S. and Sc.D. degrees from the Massachusetts Institute of Technology in 1929 and 1934, respectively. He held a Rockefeller Fellowship there and at the University of Cambridge from 1934 to 1936. At present he is research associate in the Department of Geology at Massachusetts Institute of Technology.

Alvin M. Weinberg, born April 20, 1915, received the B.S. and M.S. degrees in Mathematical Physics at the University of Chicago. He is now working for a Ph.D. degree in Mathematical Biophysics under Professor N. Rashevsky at the University of Chicago.

Here and There

Appointments and Honors

Assistant Professor Augustus Sisk, of the University of Tennessee, has been elected head of the Department of Mathematics and Physics at Maryville College, Tennessee.

★

In the New Year's honor list of King George of England the Order of Merit is conferred on Sir James Jeans, known for his work in mathematical physics and his books for the popularization of science. Sir James was Professor of Applied Mathematics at Princeton University from 1905 to 1909. Five other scientific men are members of the order at the present time. These are Sir J. J. Thomson, Sir William Bragg, Sir Frederick Gowland Hopkins, Sir Charles Sherrington and Sir Arthur Eddington. Knighthood was conferred on Dr. Robert Robinson, Professor of Chemistry at the University of Oxford.

Science

★

Dr. Enrico Fermi, Professor of Physics at the University of Rome, will join the Department of Physics of Columbia University early in January. Professor Fermi was visiting professor at the summer session in 1936.

At the Richmond meeting of the American Association for the Advancement of Science Professor E. O. Lawrence of the University of California was elected Vice President and Chairman of the Section of Physics. Dr. E. U. Condon of the Westinghouse Research Laboratories, East Pittsburgh, was elected a member of the committee for the physics section.

★

Dr. W. J. de Haas, Professor of Physics in the University of Leiden, has been elected correspondent for the section of Physics of the Paris Academy of Sciences, in succession to the late Dr. C. E. Guillaume.

★

Dr. Floyd K. Richtmyer, Professor of Physics at Cornell University and Dean of the Graduate School, has been elected Secretary of the Association of American Universities for a five-year term.

★

Prince Louis-Victor de Broglie, of the Institut Poincaré, Paris, has been elected a member in the Section of Physics, of the Royal Swedish Academy of Sciences.

Nature

Conference on Instrumentation in Process Industries

Plans have been formulated for a conference on Instrumentation in the Process Industries, sponsored by the Chemical Engineering Department of the Carnegie Institute of Technology, Pittsburgh, Pennsylvania, on March 2–3, 1939. The purpose of the conference is to bring about a better knowledge of the advance made in measurement and control in the process industries. An exhibit will be held in conjunction with the conference. Dr. C. C. Monrad of the Department of Chemical Engineering is in charge of the arrangements.

★

For March

The Operational Calculus, LOUIS A. PIPES

The Elastic and Thermoelastic Behavior of Rubber, G. GUTH

Contributed Research Papers on Measurement of Some Arc Characteristics at 1000 Atmospheres Pressure, Studies in Lubrication, IV, A Dynamic Measurement of the Elastic Electric and Piezoelectric Constants of Rochelle Salt, On the Resonant Frequency of Closed Concentric Lines, Luminescence during Intermittent Electron Bombardment, A Note on the Quantitative Spectrographic Analysis of Small Amounts of Rubidium and Potassium, Reignition of Short Arcs at High Pressures, and Convection and Conduction of Heat in Gases.

Calendar of Meetings

FEBRUARY	
13–16	American Institute of Mining and Metallurgical Engineering, New York, N. Y.
20–23	Technical Association of the Pulp and Paper Industry, New York, N. Y.
23–25	American Society of Mechanical Engineers, New Orleans, La.
23–25	American Physical Society, New York, N. Y.
23–25	Inter-Society Color Council, New York, N. Y.
23–25	Optical Society of America, New York, N. Y.
MARCH	
2–3	Conference on Instrumentation in Process Industries, Pittsburgh, Pa.
APRIL	
3–7	American Chemical Society, Baltimore, Md.
16–22	American Ceramic Society, Chicago, Ill.
26–29	Electrochemical Society, Columbus, Ohio.
27–29	American Physical Society, Washington, D. C.
MAY	
16–17	Acoustical Society of America, New York, N. Y.
22–June 8	World Automotive Engineering Congress, New York, Indianapolis, Detroit, San Francisco.
JUNE	American Physical Society.
19–24	American Association for the Advancement of Science, Milwaukee, Wis.
26–30	American Society of Testing Materials, Atlantic City, N. J.
JULY	
6–8	Sixteenth Colloid Symposium, Stanford University, Calif.
SEPTEMBER	
4–12	International Congress of Mathematicians, Cambridge, Mass.
4–15	International Union of Geodesy and Geophysics, Washington, D. C.
11–15	American Chemical Society, Boston, Mass.
DECEMBER	
27–Jan. 2	American Association for the Advancement of Science, Columbus, Ohio.

★

One of the Physical Songs of the Cavendish Society, Cambridge, England

Taken from the Program of the Toronto Meeting of the APS and AAPT

IONS MINE

Air: "Clementine."

1. In the dusty lab'ratory,
   'Mid the coils and wax and twine,
   There the atoms in their glory
   Ionize and recombine.

Chorus: Oh my darlings! Oh my darlings!
Oh my darling ions mine!
You are lost and gone for ever
When just once you recombine!

2. In a tube quite electrodeless,
   They discharge around a line,
   And the glow they leave behind them
   Is quite corking for a time.

3. And with quite a small expansion,
   1.8 or 1.9,
   You can get a cloud delightful,
   Which explains both snow and rain.

4. In the weird magnetic circuit
   See how lovingly they twine,
   As each ion describes a spiral
   Round its own magnetic line.

5. Ultraviolet radiation
   From the arc or glowing lime,
   Soon discharges a conductor
   If it's charged with minus sign.

6. α-rays from radium bromide
   Cause a zinc-blende screen to shine
   Set it glowing, clearly showing
   Scintillations all the time.

7. Radium bromide emanation,
   Rutherford did first divine,
   Turns to helium, then Sir William
   Got the spectrum—every line.

Contributed Original Research

Double Refraction and Change in Length of Certain Plastics*

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(Received July 27, 1938)

The change in length of a Vinylite plastic under constant load has been measured, as well as for the succeeding recovery. The change in color of the light transmitted through crossed Polaroid plates shows large change in the birefringence. The further study is made with uniform length increments at constant temperature using Mach's method to measure the birefringence, both for stretching and recovery. The data are sufficient to give a measure of the dispersion in birefringence and of a Poisson's ratio of 0.50.

I. Progressive Stretch and Recovery

SOME of the new plastics exhibit striking effects in polarized light, and while there are those well suited to use in photoelastic models,[1] others possess such time effects that they cannot be used for this purpose. Nevertheless these time effects are of interest and must be considered in theories of structure and molecular behavior. The results reported below were obtained with strips of plasticized "Vinylite" XYSG obtained through the courtesy of the Carbide and Carbon Chemicals Corporation. This flexible material is very sensitive photoelastically, and when only slightly bent or stretched between crossed Polaroid plates shows quite clearly the interference colors.

Since the colors in every instance show rapid changes with time an attempt was made to obtain quantitative measurements on a simple case of stretching under constant load. The experiment at the outset is fascinating when watched through Nicols or Polaroid plates. The material is slightly doubly refracting in its original state, so that it appears nearly white observed in this way. As soon as the load is applied the transmitted color changes rapidly, depending, of course, upon the load, and may in the first two or three minutes pass through several orders. The change gradually becomes less rapid, though in no case has it become certainly zero, even after a period of several weeks. It is then evident that to follow the length change closely an accurate temperature control is necessary, and no such control was used in the experiments described.

When the load is removed the strip immediately begins to shorten, the color change is very rapid, apparently much more so than when the load is first applied, and in course of time becomes so slow that watching for change is again tedious. Some of the plastics retain a more or less permanent set, but the Vinylite under the various loads used has always returned practically to its original length, the last stages of the return being extremely slow. For this reason the data reported are those taken on a fresh strip which had not previously been used to satisfy the usual laboratory curiosity.

The strip in question had an original length of 7.60 cm between the fiducial marks followed by the measuring microscopes. The original width was 0.81 cm and thickness 0.075 cm. Properly a correction for end effects should be made, but this was not done, since the nonhomogeneous

* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.
[1] A. G. Solakian, Mech. Eng. **57**, 767 (1935).

strain extended only a few millimeters from the clamps. The load was 200 grams. The strip was hung in a parallel beam of white light between Polaroid plates at angles $-45°$ and $+45°$ to the vertical so that the color changes could be observed during the intervals between length measurements.

The curves (Fig. 1) give the run of $\Delta L/L_0$ with time, both for stretch and recovery, with a double time scale to show how the length changed at the start. The recovery curve after the first two days is not shown, since the temperature of the laboratory went outside the two-degree range of the rest of the experiment. Furthermore, the case reported was not carried beyond the 10-day period as it was found that the small temperature range of the laboratory was nevertheless sufficient to affect the measurements.

On applying the load for this particular case the transmitted color through the second Polaroid had passed through all the first-order colors within the first minute, was in the red again by the fifth minute, and at the end of the experiment was nearly in the third red. On the recovery the change was more rapid at the start, and by the end of the first minute the color had returned to the first blue. At the end of one hour

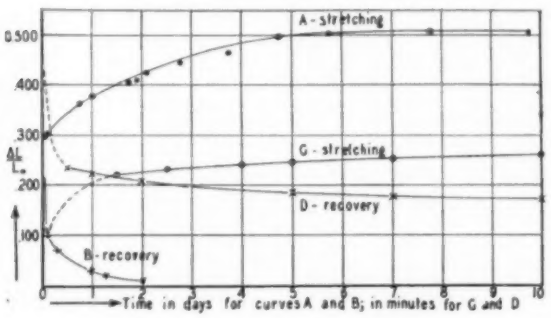

FIG. 1.

it was in the first orange, after this the change back to the original white was very slow.

Attempts to fit these curves to the better known equations[2] have been unsatisfactory, and it seems more suitable at this time to present the data for consideration.

Some of the duPont plastics are even more sensitive than is Vinylite, and although they are

[2] Cf. R. Houwink, *Elasticity, Plasticity and Structure of Matter* (University Press, Cambridge and Macmillan, New York, 1937).

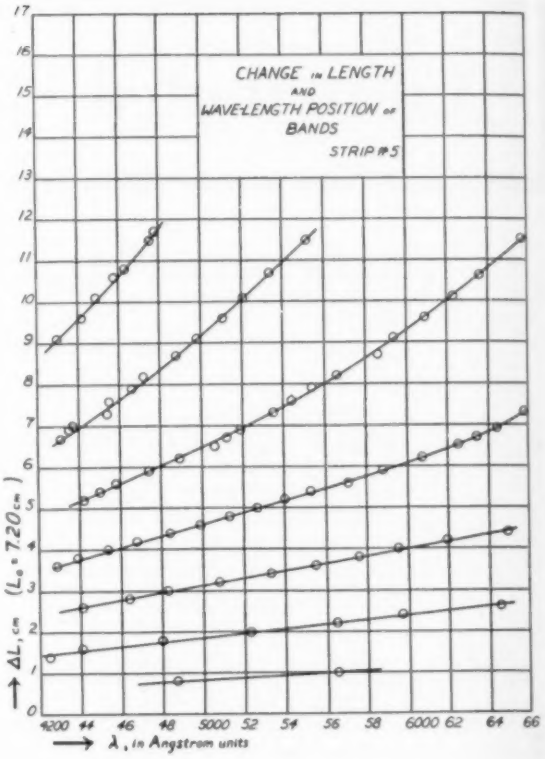
CHANGE in LENGTH
AND
WAVE-LENGTH POSITION of
BANDS
STRIP #5
ΔL, cm (L₀ = 7.20 cm)
4200 44 46 48 5000 52 54 56 58 6000 62 64 66
λ, in Angstrom units

FIG. 2.

even softer, nevertheless possess remarkable recovery. While thin sheet rubber exhibits interesting color changes during stretching, its elastic behavior is in general better known than that of the materials used.

II. Birefringence and Uniform Change in Length

Spectroscopic examination of the transmitted light showed that, in spite of the fact that the Vinylite resin is hardly good optical material, the spectrum gave fairly satisfactory dark bands when the material was sufficiently stretched. Consequently it seemed worth while to see what information these bands could give. If the specimen is mounted vertically between crossed Nicols whose principal planes are at 45°, then the phase change between vertical and horizontal vibration components is $\delta = 2\pi d(n_1 - n_2)/\lambda$, where d is the thickness of the material, λ the wavelength, and $n_1 - n_2$ the difference between the two indices of refraction or the birefringence, which will here be denoted B. If the Nicols are

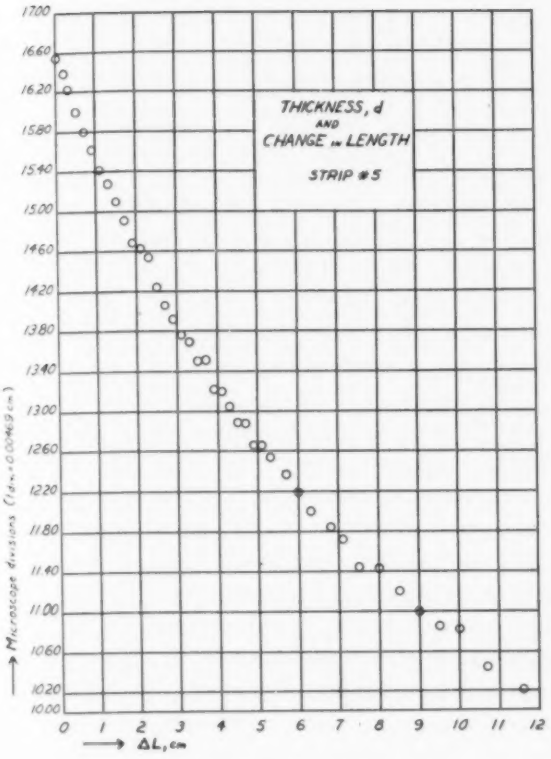

FIG. 3.

crossed and there is no birefringence ($B=0$), there is of course no light transmitted, and there will be no light transmitted, at a wave-length λ, when B is not 0, if for that wave-length $\delta=2\pi$ or any whole number multiple thereof. Hence if in the spectrum a black band of the first order be located, $\lambda_1=Bd$, or in general, $k\lambda_k=Bd$, where k is the order of the band. If then d can be measured at the same time with the wave-length position of the band, the value of B can be determined. This principle probably was first used by Mach, later by Kundt and others, and the idea has been developed further, notably by Filon.[3]

The studies under constant load showed the difficulty of obtaining satisfactory simultaneous measurements of λ and d, each one of which requires a number of minutes. It was therefore decided to carry out these measurements with constant increments of length. The specimen was mounted in a vertical brass chamber with its lower end fixed and its upper end clamped to a rod on which a good millimeter thread had been cut. By turning the nut at the top of the chamber one full turn the specimen was thus extended one millimeter. The Nicols were set at openings on opposite sides of the chamber, and at the same height at right angles to the axis of the Nicols was set the microscope for thickness measurements. The whole chamber was wound with copper tubing through which a stream of water flowed constantly to maintain a fixed temperature. In order to ensure a reproducible operation the nut was given one whole turn at regular intervals, a five-minute period being the one in the case for which the data are given. There is very little difference between the results for a five-minute interval and for a two-minute interval. The light transmitted by the analyzer was

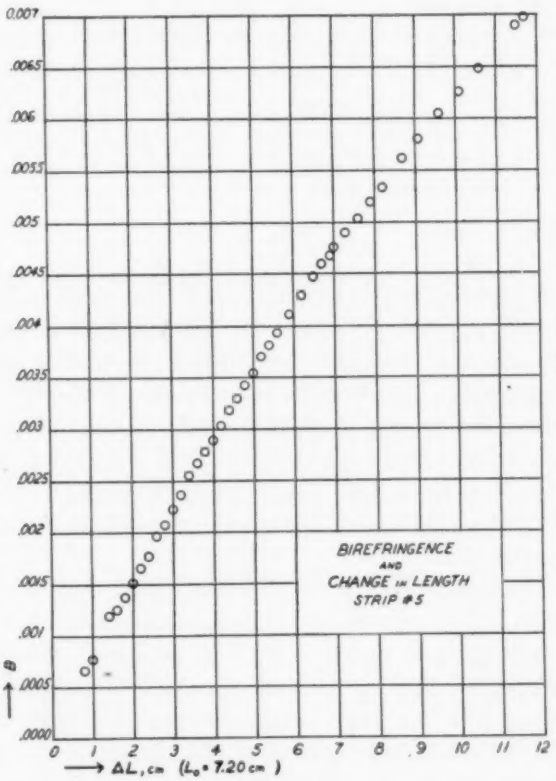

FIG. 4.

received on the slit of a Hilger wave-length spectroscope.

In Fig. 2 the position of the bands observed is plotted against the number of turns of the nut, ΔL. It is seen from the curves that as the stretching progressed the bands moved across

[3] E. G. Coker and L. N. G. Filon, *Photoelasticity* (Cambridge University Press, 1931), p. 22.

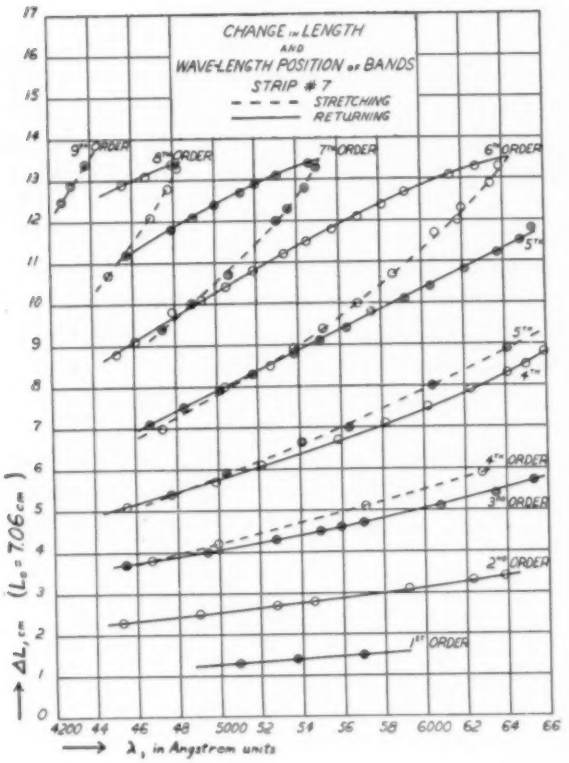

FIG. 5.

From these data then the values of B were computed and the results are plotted against ΔL in Fig. 4. Examination of this curve shows that at the outset it is very nearly a straight line. The fact that this straight line does not pass through the origin indicates the presence of some initial birefringence, probably also a slight error in setting the nut originally to a position agreeing with no extension of the strip. Properly then for a determination of the strain, the zero point should be taken from the curve rather than from the initial position of the nut.

The values of B on this curve were computed from the average of the values of $k\lambda_k$, $(k+1)\lambda_{k+1}$ and $(k+2)\lambda_{k+2}$ for the various d settings. This is a reasonable procedure, as the positions of the bands near the ends of the spectrum are not as accurately determined as are those in the central region. However, it is possible from the curves of Fig. 2 to obtain more definite information concerning the dispersion. If on this curve we locate the values of ΔL for, say, $\lambda = 4500$A, and take from Fig. 3 the corresponding values of d, we can then compute for that wave-length the run of B with ΔL, and by similar computations

the spectrum from blue to red, the third-order band appearing before the second had disappeared, and later three orders were present at the same time. The simultaneous existence of two or more bands gave a check on the accuracy of the birefringence measurements, since for two such bands, $k\lambda_k = B_1 d$ and $(k+1)\lambda_{k+1} = B_2 d$, or $k\lambda_k/(k+1)\lambda_{k+1} = B_1/B_2$, independent of measurements of d.

The measurement of thickness, d, was made with micrometer microscope of suitable magnification. Here the real problem is to know the thickness where the light is passing through the material. Since the material certainly has not optical surfaces and is so soft that the cut edge shows numerous minor variations, it seemed most reasonable to measure the thickness directly as carefully as possible, and then to take the value for computing from the plotted curve. Hence for the data on d shown in Fig. 3, the smooth curve, indicated by the observations, is assumed to give the most reliable path distance through the material.

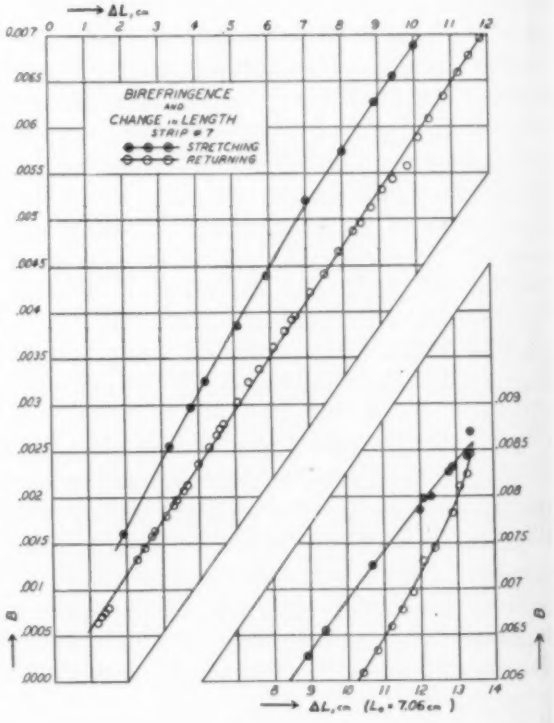

FIG. 6.

for 5000, 5500, 6000 and 6500A, obtain the data which show very small but definite differences, the values of B for the shorter wave-lengths being larger than those for longer wave-lengths. The order of magnitude of this difference at $\Delta L = 10$ cm can be expressed as $B_{4500} - B_{6500} = 0.001$. Measurements on width and thickness indicated that during the stretching they decrease proportionally. Width was not measured for the strip in the constant temperature chamber, but it was assumed that the cross-sectional area would be proportional to d^2. With this assumption the data show that if there is a volume change under the conditions of stretching, it is certainly less than one percent. This of course means a Poisson's ratio of 0.50, which has been found to hold for rubber.

A further test was made to include both the increase in length and the subsequent decrease. A fresh strip was extended nearly 200 percent by one-millimeter steps every two minutes and then shortened the same way. The curves for this experiment are Figs. 5 and 6. It is obvious that the return differs decidedly from the stretching. The strip finally became slack before the nut had reached the zero position, in other words the two-minute interval was too short for the strip to keep up with. In all of these experiments the stretching was in the direction of calendering.

Birefringence measurements on rubber have generally been made on elastic rather than plastic rubber and have followed a somewhat different procedure,[4] yet they seem to indicate a similar behavior in many respects. The Vinylite resin, like rubber, is raised in temperature by stretching, and when under constant load shortens when heated. When heated, either with constant load or constant length, the birefringence decreases. When stretched and left standing at constant temperature the birefringence decreases. Measurements on these changes are in progress.

[4] W. E. Thibodeau and A. T. McPherson, Nat. Bur. Standards J. Research **13**, 887 (1934).

The Viscosity of Pennsylvania Oils at High Pressure

R. M. Dibert, R. B. Dow and C. E. Fink
School of Chemistry and Physics, The Pennsylvania State College, State College, Pennsylvania
(Received October 15, 1938)

Viscosity-pressure-temperature data are presented for six Pennsylvania oils which differ in chemical and physical properties at atmospheric pressure. This study is a continuation of one reported previously and employs the same apparatus and experimental method. While the pressure coefficient of viscosity increases in general with molecular weight, the relationship seems to be significant only for fractions of a narrow boiling range from a single oil. The mean temperature coefficient of viscosity, between 100° and 210°F, however, increases with molecular weight at all pressures for all the oil samples.

A PREVIOUS paper[1] presented viscosity-pressure-temperature data for three mineral oils refined from California, Oklahoma, and Pennsylvania crudes. This paper is the result of a similar study made on six Pennsylvania oils, an investigation undertaken to study the characteristics of such oils in more detail than has been made hitherto. Particularly, the viscosity at several pressures and temperatures has been studied in relation to the boiling point, temperature coefficient of viscosity, and molecular weight.

Of the six oils, numbers 1, 2 and 4 were commercial Pennsylvania products. No. 1 was a "bright stock" of high molecular weight, No. 2 a "neutral" of low molecular weight, and No. 4 a highly refined oil, solvent extracted with Chlorex. Oils 3, 5 and 6 were narrow boiling cuts fractionated from oil No. 4 in the Petroleum Refining Laboratory of this school. Table I is a summary of the characteristics of the oils at atmospheric pressure.

[1] R. B. Dow, J. App. Phys. **8**, 367 (1937).

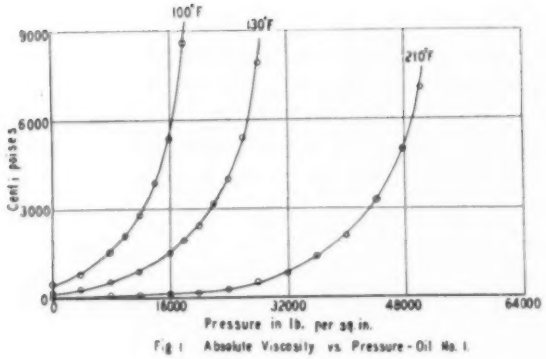

Fig 1 Absolute Viscosity vs. Pressure - Oil No. 1.

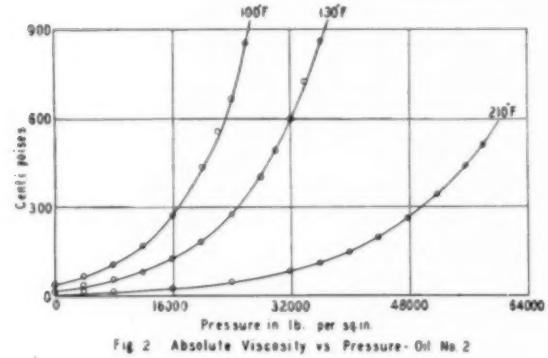

Fig 2 Absolute Viscosity vs. Pressure - Oil No. 2

The pressure effects were studied in the High Pressure Laboratory of the Department of Physics and the apparatus was similar to that used in the previous study.[1] A rolling-ball viscometer, the temperature of which was regulated by a thermostated bath, was used in measuring viscosity over a pressure range of 58,000 lb./in.^{2} at temperatures of 100°, 130° and 210°F. A new calibration curve was necessary since the angle of tilt was less than that used in the previous study. Consequently, for long roll times the absolute coefficient of viscosity, μ, is given by (See Eq. (3) of previous paper):[1]

$$\mu(7.36-\rho)/155.5T.$$

As before, ρ is the density of the oil in absolute units and T is the corrected roll time in seconds. The characteristics of the calibration curve were so similar to the one used previously that it is unnecessary to reproduce it here.

TABLE I. *Data on Pennsylvania oils at atmospheric pressure.*

Oil No.	Characteristics	Boiling-Point Range: In degrees F 10 mm Hg absolute pressure	Molecular Weight	Viscosity Index	Density (g/cm^3): 100°F	130°F	210°F
1.	Commercial "Bright Stock"	583–664	706	102	0.879	0.869	0.841
2.	Commercial "Neutral"	459–588	368	100	.860	.849	.821
3.	High Boiling Point Fraction From Oil 4	517–534	370	117	.841	.830	.801
4.	Solvent Refined "Neutral"	450–567	352	116	.838	.827	.798
5.	Medium Boiling Point Fraction From Oil 4	482–504	342	122	.836	.825	.796
6.	Low Boiling Point Fraction From Oil 4	453–478	310	127	.832	.821	.792

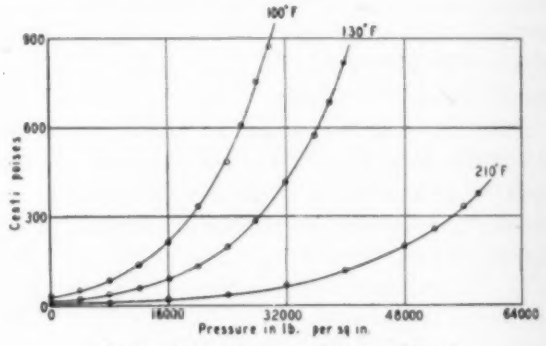

Fig 3 Absolute Viscosity vs. Pressure - Oil No. 3

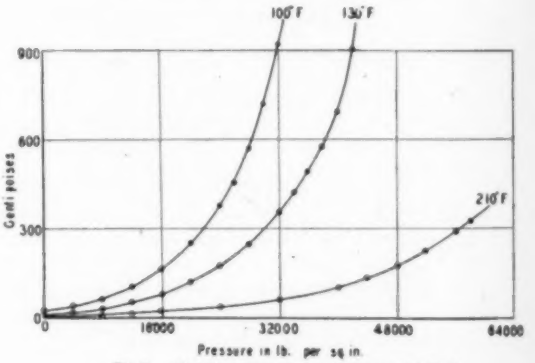

Fig 4 Absolute Viscosity vs. Pressure - Oil No. 4

Data and Discussion

The viscosities, measured independently by the capillary pipette and rolling-ball methods, are given in Table II. The results by both methods agree closely, the worst disagreement being about eight percent at 210°F for a viscosity of about four centipoises.

The viscosities at higher pressures are recorded in Figs. 1–6.

On examining the data shown on curves in Figs. 1–6, it will be observed that the oil having

the higher coefficient of viscosity at atmospheric pressure at any temperature will likewise have the greater coefficient under pressure. While the viscosities of the fractions, oils 3, 5 and 6 increased uniformly with the molecular weights, those of the commercial oils did not, a result that indicated that the relation between viscosity and molecular weight is of significance only when fractions of a narrow boiling range are used.

Table III contains computations of the mean temperature coefficient of viscosity, calculated over the range 100° to 210°F from the pressure data.

While the viscosity index at atmospheric pressure of these oils did not seem to be directly related to the viscosity at various pressures and temperatures, the mean temperature coefficient of viscosity did seem to be related to the molecular weight, both increasing together with increase of pressure. For each oil, it was found that the temperature coefficient of viscosity increased with pressure as shown in Table III, a result which is normal for pure liquids.

These studies are being continued at present with the intention of mapping in greater detail the viscosity characteristics of lubricating oils over a wide range of experimental conditions. In conclusion, the authors acknowledge the interest and cooperation of Dr. M. R. Fenske. They are indebted to the Pennsylvania Grade Crude Oil Association for partial support of this program.

TABLE II. *Viscosities at atmospheric pressure (centipoises).*

Oil Number	100°F		210°F	
	Capillary Pipette	Rolling Ball	Capillary Pipette	Rolling Ball
1.	406.00	415.00	25.90	26.05
2.	34.40	34.05	4.82	4.90
3.	26.20	26.90	4.01	4.33
4.	23.40	23.00	3.75	3.87
5.	20.00	19.65	3.62	3.54
6.	14.20	14.45	2.70	2.87

TABLE III. *Mean temperature coefficient of viscosity 100°–210°F.*

Oil No.	14.2 lb./in.²	10,000 lb./in.²	18,000 lb./in.²	30,000 lb./in.²	Molecular Weight	Viscosity (centipoises) 100°F	Viscosity (centipoises) 210°F
1.	0.00850	0.00870	0.00890		706	415	26
2.	.00778	.00812	.00835		368	34	5
3.	.00762	.00810	.00829	0.00852	370	27	4
4.	.00757	.00796	.00803	.00844	352	23	4
5.	.00745	.00777	.00796	.00848	342	20	4
6.	.00730	.00764	.00794	.00828	310	14	3

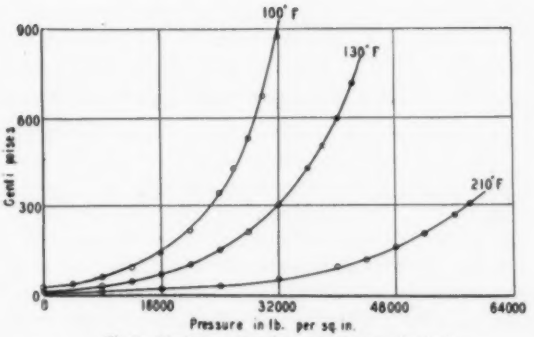

Fig. 5 Absolute Viscosity vs. Pressure - Oil No. 5

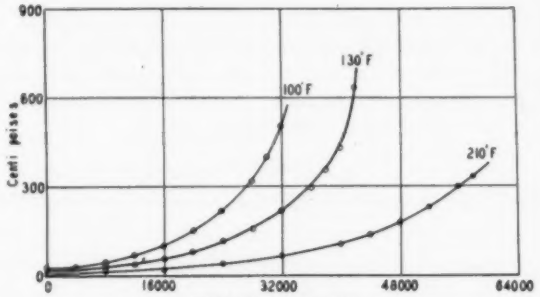

Fig. 6. Absolute Viscosity vs. Pressure - Oil No. 6

Light Output and Secondary Emission Characteristics of Luminescent Materials

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Research and Engineering Department, RCA Manufacturing Company, Inc., Harrison, New Jersey
(Received October 25, 1938)

Light output and screen potential data are given for zinc sulphide, calcium tungstate, willemite and several other silicate phosphors sprayed (from suspension) on Nonex glass and bombarded by electrons. The bombarding voltage range from 0.5 to 10 kilovolts is covered. The limiting potentials of willemite and other zinc silicate screens fall in the range from 6 to more than 10 kilovolts depending on the conditions of exhaust and preparation. The limiting potentials of two zinc sulphides fall in the range from 6 to 9 kilovolts while calcium tungstate has a limiting potential usually less than 5 kilovolts. Light output as a function of screen voltage over the range from 0.5 to 10 kilovolts is shown to follow a law of the form:

$$L = Af(i)V^n,$$

where L is intensity of light, i is current density, V is electron energy at the screen in kilovolts, and A is a constant. The data show that $n<2$ for willemite or other zinc silicates, $n=2$ for sulphides, and $n>2$ for calcium tungstate and a fused layer of willemite. Analysis and comparison of the light output current density characteristic and the persistence characteristic shows that a definite relation exists between them in the case of silicate phosphors. The function of current density may be represented by

$$f(i) = Ai(1-(a/(a+b))e^{-\alpha/Bi}-(b/(a+b))e^{-\gamma/Bi}).$$

The constant α and γ are those appearing in the persistence characteristics of the material as:

$$L = (a/(a+b))e^{-\alpha t}+(b/(a+b))e^{-\gamma t},$$

a formula which expresses well the persistence of silicate screens.

INTRODUCTION

THE fundamental processes governing both the excitation and emission of radiation in luminescent materials are of importance in the design of cathode-ray tubes. This is particularly true where it is desirable to operate the tubes at high values of light output which require high current densities and high voltages, as in the case of the projection Kinescope and oscilloscope tubes designed for observation of high speed transients.

It is well known that the luminescent screen of cathode-ray tubes is a relatively good insulator and therefore maintains its potential largely by secondary emission. Some measurements of the potential at which a willemite screen floats with respect to the collector potential in the low voltage range to 1500 volts have been reported by Brown.[1] Data on light output of zinc sulphide on metal and on glass over the voltage range from 30 to 80 kilovolts have been given by K. Scherer and R. Rubsaat.[2] These data indicate that the luminescent material on glass does not follow the acceleration potential in this range. Further screen potential data over the range of voltage from about 1000 to 10,000 volts have been reported by Nottingham.[3] Some data on the limiting potential of glass, willemite and calcium tungstate are given by Von C. Hagen and H. Bey.[4] Nelson[5] has recently reported on a method of measuring screen potentials which is readily adaptable to most any type of cathode-ray tube with a screen on the glass wall. The method of Nelson is referred to as the glass-conduction method because it involves heating the glass support of the screen to a temperature at which the glass support becomes sufficiently conducting to have a resistance low compared with the grid resistance of an electrometer tube which is used for measurement of the screen potential. The details of the method are described by Nelson[5] and therefore will not be repeated here.

The glass-conduction method of screen-potential measurement was used to extend the preliminary results of Nelson to a number of luminescent screen materials such as silicates, sulphides, and tungstates. Measurements were made on a number of tubes with screens of the same silicate, and a number of tubes with screens

* Department of Physics, Clark University, Worcester, Massachusetts.

[1] T. B. Brown, J. Opt. Soc. Am. **27**, 186 (1937).

[2] K. Scherer and R. Rubsaat, Archiv. f. Electrotechnik **31**, 821 (1937).

[3] W. B. Nottingham, J. App. Phys. **8**, 762 (1937).

[4] Von C. Hagen and H. Bey, Zeits. f. Physik **104**, 681 (1937).

[5] H. Nelson, J. App. Phys. **9**, 592 (1938).

of different materials in the same envelope. Data were taken on the light output as a function of beam-current density with both a stationary and a scanning spot; also, as a function of screen potential at constant-current density for different types of luminescent screens. An analysis was made of the light-output, current-density characteristics and of the persistence characteristics of willemite, zinc sulphide, and calcium tungstate.

Data and Results

The curves in Fig. 1 show the type of screen-potential data obtained on different cathode-ray tubes of the same type using the same lot of a yellow zinc beryllium silicate (phosphor No. 3) applied by the same process to give a screen of the same thickness controlled within reasonably close limits. The tubes were exhausted under standard factory conditions using the same pump position. A barium-aluminum getter pellet was flashed in the neck of the tube near the cathode just before seal-off from the pumps. Similar data were taken on a number of other tubes using the same screen material but pumped on different exhaust positions. The screen potential of all measured tubes varied somewhat more between eight and ten kilovolts on anode No. 2 than is shown in Fig. 1. A correlation between exhaust position and screen potential was noted and is probably due to slight differences in pressure during the getter flash which resulted in different amounts of barium on the screen for tubes pumped on different positions. However, the data of Fig. 1 are representative of the results which can be obtained on different tubes with the same screen material made under closely controlled conditions. Widely different results have been obtained on the same luminescent material with similar screens in tubes processed by different methods.

It is realized that secondary-emission characteristics are influenced widely by slight contaminations of the screen surface and therefore no claim is made for the accuracy of results, such as shown in Fig. 1 illustrating characteristics of a clean screen of zinc beryllium silicate. A slight contaminating influence of barium from the getter flash has been indicated by results mentioned above. Furthermore, as will be discussed later, when screens may be slightly contaminated with barium, distinct differences in secondary-emission characteristics have been observed among screens of different thickness and among different types of luminescent materials in tubes made under nearly identical conditions. In general, as the anode No. 2 potential or the velocity of electrons bombarding the screen, is increased, the screen potential increases to a definite value above which it will increase only slightly regardless of the magnitude of the anode No. 2 voltage. This maximum value of potential is defined as the limiting potential of the screen, and is usually expressed with respect to cathode potential.

The influence of screen thickness, for two screens of widely different thickness, on the secondary-emission characteristics is shown by the curves in Fig. 2. The thin screen with a white-light transmission of 86 percent (a bare dusting of phosphor No. 3 on Nonex glass) shows a limiting potential of about 5000 volts or only about 2000 volts above that of Nonex glass as reported by Nelson,[5] while the thick screen with a white-

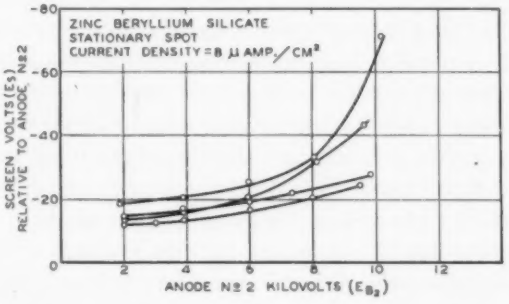

Fig. 1. Luminescent screen potential characteristic curves of four cathode-ray tubes with the same phosphor screens, manufactured and exhausted under closely controlled factory conditions.

light transmission of 25 percent (about as heavy a coating of phosphor No. 3 as could be sprayed onto the glass without a binder) shows no tendency to reach a limiting potential up to 10,000 volts.

As Nelson[5] has pointed out, the secondary-emission characteristic of luminescent screens, especially thin ones, is a composite of that of glass and that of the luminescent material because the luminescent material does not cover the glass completely. Therefore, wide differences in secondary-emission characteristics may be expected to

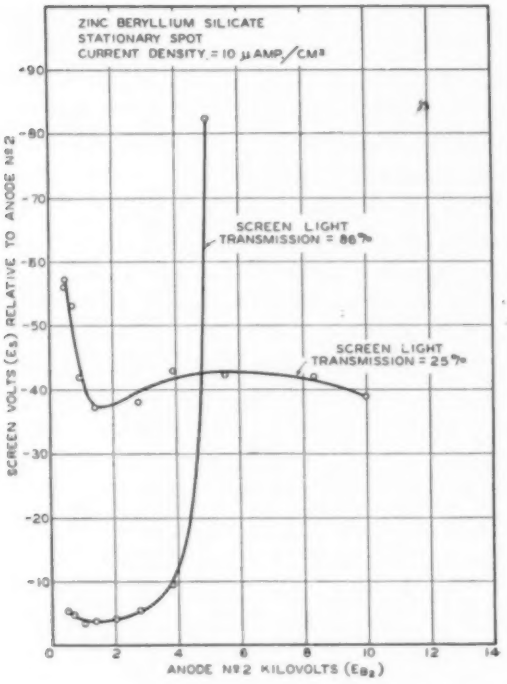
ZINC BERYLLIUM SILICATE
STATIONARY SPOT
CURRENT DENSITY = 10 μ AMP./CM²
SCREEN LIGHT TRANSMISSION = 86%
SCREEN LIGHT TRANSMISSION = 25%
SCREEN VOLTS (Es) RELATIVE TO ANODE №2
ANODE №2 KILOVOLTS (Eb2)

FIG. 2. Luminescent screen potential characteristic curves for sprayed silicate screens of widely different thickness in the same envelope.

occur when measurements are made on screens of different thickness and on different base materials.

Figure 3 shows the potential characteristics of a fused willemite layer. This was made by fusing the material in air to a platinum backing piece having a thickness of about 0.001 inch. This backing piece was sufficiently thin so that no visible cracks appeared in the surface of the phosphor on cooling. It showed various anomalous properties as regards light output which will be discussed later. Its limiting potential was about 6000 volts, and was in agreement with Nottingham's[3] data. The reason for this agreement may be due to a change in surface because of the fusion in air, or simply because this sample was exhausted under conditions comparable to his. When the getter was flashed after vacuum-heating the material, the vacuum was 3×10^{-7} mm of mercury or better.

Figure 4 shows the potential characteristics of four different silicates all in the same envelope, one of which has considerable barium in its composition. Fig. 5 is a similar plot for different zinc sulphides of foreign origin. Sample *A* and sample *B* were used in one tube and a sample of calcium tungstate was used in another tube. It is notable that all the silicates, although of widely different compositions, have similar characteristics, but that these differ from those of the sulphides, and also from the characteristic of calcium tungstate.

Incidentally, it must be mentioned that the tungstate showed a marked change in both light output and secondary-emission characteristics at the higher voltages. This change was caused by heating the glass. The limiting potential decreased a few hundred volts with respect to cathode potential and the light dropped simultaneously to about a fifth of its value. Calcium tungstate is the only material which showed any measurable effect of this nature, although it is suspected that the light output of the sulphides was affected by the high energy input of the beam at highest voltages. There was, however, no noticeable effect on the screen potential.

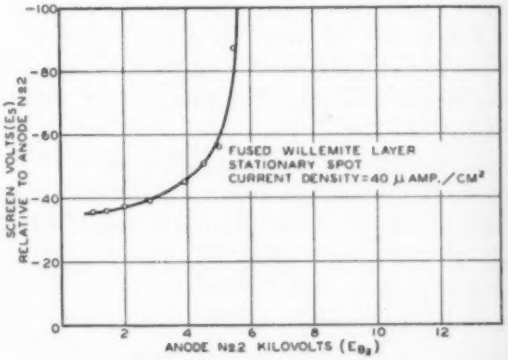
FUSED WILLEMITE LAYER
STATIONARY SPOT
CURRENT DENSITY = 40 μ AMP./CM²
SCREEN VOLTS (Es) RELATIVE TO ANODE №2
ANODE №2 KILOVOLTS (Eb2)

FIG. 3. Luminescent screen potential characteristic curves for a fused willemite layer in a tube exhausted under the best high vacuum conditions.

LIGHT MEASUREMENTS

It is customary in engineering practice to designate the characteristics of a fluorescent material at a given voltage in terms of its brightness per square centimeter per watt (lumens per steradian per square centimeter per watt) or in terms of horizontal candlepower per watt. Since the light output may not be linear with current, this method of indicating the characteristics of the material is not necessarily desirable, particularly from the point of view of one attempting to correlate the characteristics with other physical and chemical properties of the substance.

In the figures which follow, the form of the light-output characteristics of green willemite (phosphor No. 1) and yellow zinc beryllium silicate (phosphor No. 3) are compared with blue calcium tungstate (phosphor No. 5) and two sulphides (a blue zinc sulphide and a yellow zinc cadmium sulphide), both for constant current and constant voltage, under the steady bombardment of a stationary beam of relatively low current density and for a focused scanned spot of high current density. These phosphors will be referred to later as willemite, zinc beryllium silicate, calcium tungstate, zinc sulphide and zinc cadmium sulphide, respectively.

In Fig. 6 are shown constant-voltage curves with a stationary beam of low current density. These curves illustrate the nature of the dependence of light on current density over a good range of current density. The scale of ordinates is different for each material and is intended merely to show that the zinc beryllium silicate is not linear except for values of current density less than about ten microamperes per square centimeter (data of Nottingham[3] and of Brown[1] indicate that the departure from linearity takes place somewhere between one and five microamperes per square centimeter for willemite) while the zinc sulphides and calcium tungstate are linear up to much higher values of current densities, i.e., to about 100 microamperes per square centimeter.

In Fig. 7 are shown constant-voltage curves with a scanning beam of high current density. These curves illustrate the nature of the dependence of light on current density over a considerable range of current density. The scale of ordinates is different for each material. The curve for the willemite in Fig. 7 is to be compared with that of the zinc beryllium silicate in Fig. 6, and the curve for the zinc cadmium sulphide in Fig. 7 is to be compared with that of the zinc sulphide in Fig. 6, because the silicates as a class have similar characteristics and so do the sulphides. Under scanning conditions, each material becomes nonlinear for values of average current density above a few microamperes per square centimeter over the scanned area, or for values of spot current density above a few amperes per square centimeter. Both the sulphide and the tungstate show a greater departure from linearity than does the willemite at the higher values of current density.

The sulphides and tungstate have an enormously shorter persistence than the silicates. The intensity drops to one percent of its initial value in about 10^{-5} second for the tungstate, 10^{-3} second for the sulphides, and in about 5×10^{-2} second for silicates. Therefore, when a phosphor is under a steady bombardment corresponding to a given value of current density, the degree of

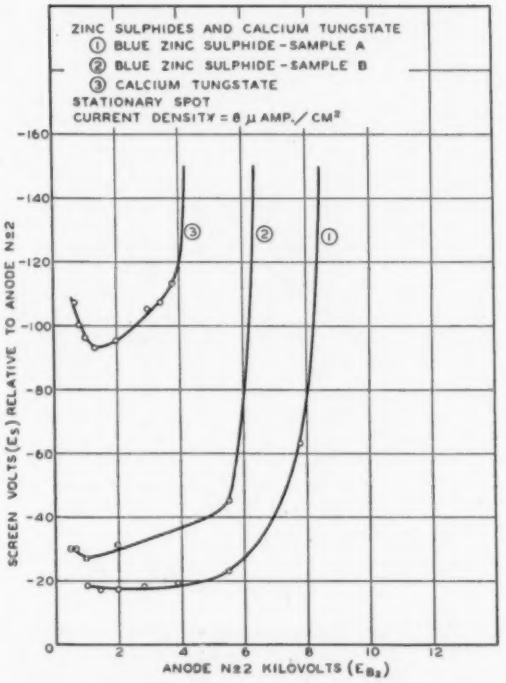
ZINC SULPHIDES AND CALCIUM TUNGSTATE
① BLUE ZINC SULPHIDE - SAMPLE A
② BLUE ZINC SULPHIDE - SAMPLE B
③ CALCIUM TUNGSTATE
STATIONARY SPOT
CURRENT DENSITY = 8 μ AMP./CM²
SCREEN VOLTS (Es) RELATIVE TO ANODE №2
ANODE №2 KILOVOLTS (EB2)

FIG. 5. Luminescent screen potential characteristic curves for two sulphides in the same envelope.

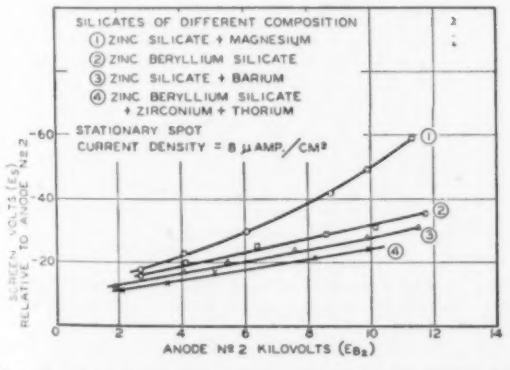
SILICATES OF DIFFERENT COMPOSITION
① ZINC SILICATE + MAGNESIUM
② ZINC BERYLLIUM SILICATE
③ ZINC SILICATE + BARIUM
④ ZINC BERYLLIUM SILICATE + ZIRCONIUM + THORIUM
STATIONARY SPOT
CURRENT DENSITY = 8 μ AMP./CM²
SCREEN VOLTS (Es) RELATIVE TO ANODE №2
ANODE №2 KILOVOLTS (EB2)

FIG. 4. Luminescent screen potential characteristic curves for four silicates of different composition in the same envelope. (The magnesium, barium zirconium and thorium are considered to be present as silicates or oxides in the silicate compounds.)

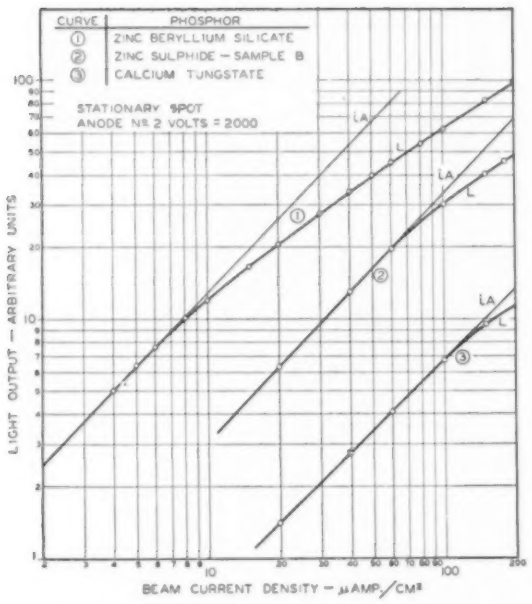

FIG. 6. Light output characteristic curves for a stationary electron spot bombarding several phosphors. Departure of the curve from the 45-degree line indicates departure from linearity.

excitation or number of excited centers per unit volume will be much higher for the silicates than for the sulphides or tungstate. The higher degree of excitation for a given current density leads to the appearance of light-output saturation which occurs at lower values of current density.

The fact that the sulphides and tungstate under scanning conditions show a greater departure from linearity of light output with current density at the higher values of current density than the silicates, may be a result of the lower activator content of the sulphides and tungstate, or to their greater susceptibility to loss of light output with increased temperature.

Before the light-output curves which follow are discussed, it should be mentioned that the light was measured from the bombarded side of the screen with a stationary spot whose area was one square centimeter, and from the opposite side of the screen for the focused spot scanning an area of 50 square centimeters.

Figure 8 shows the light output as a function of screen voltage, at constant-current density, for a number of fluorescent materials. Over almost the entire range of voltages and for all materials, the curves can be fairly well represented by a law of the form

$$L = Af(i) \cdot V^n,$$

where L = light intensity
i = current density
V = electron energy at the screen
A = a constant.

The exponent n is the slope of the log-log plot, since

$$\log L = \log [Af(i)] + n \log V.$$

The two silicates (zinc beryllium silicate and willemite) seem to have practically the same slope; hence, the same exponent in the above law. The two zinc sulphides (sample A and sample B) applied by spraying without binder

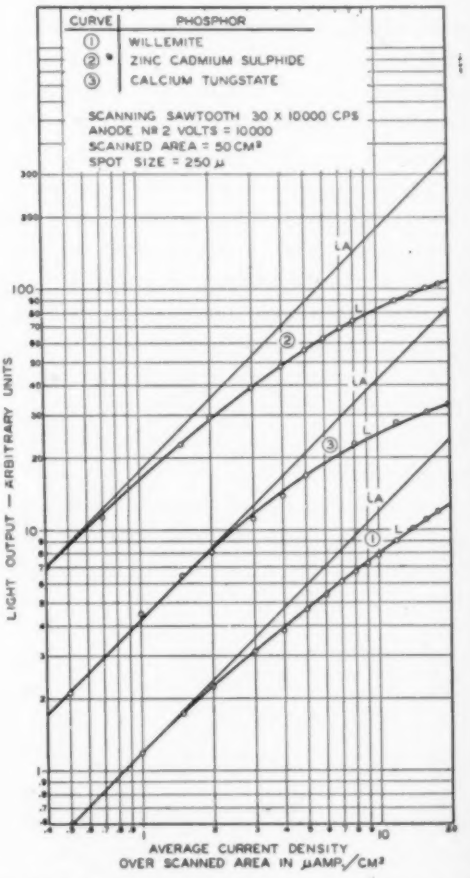

FIG. 7. Light output characteristic curves for a spot which completely scans a given area of several phosphor screens.[6] Note the widely different departures from linearity for the different materials, as indicated by the difference between the curve and the 45-degree line.

[6] Unpublished data of L. F. Swedlund of the Research and Engineering Department, RCA Manufacturing Company, Inc., Harrison, New Jersey.

also have the same exponent (after sample *A* was aged). The exponent for the sulphides is practically two, which is larger than that for the silicates, but somewhat less than that for the calcium tungstate. It is interesting that the exponent for the sample *A* zinc sulphide was considerably greater when measured with ascending voltage, than when measured with descending voltage. This effect was caused by the action of the electrons at the highest voltage, where the light was observed to decay to a stable value. When the exponent was measured with descending voltage, the slope was observed to be almost exactly two. This difference in slope of the curve indicates that some fundamental change has occurred by the material under bombardment. It may have been a partial decomposition although no gas liberation was observed.

Figure 9 shows that the slope of the curve is somewhat different for the sample *B* zinc sulphide when the current density is high than when the current density is low. The higher value of current density was well off the linear portion of the light *versus* current-density characteristic.

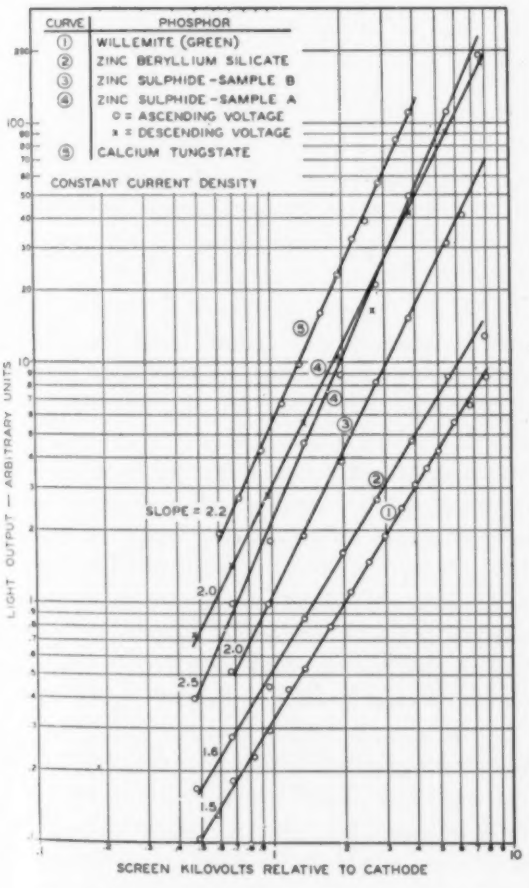

FIG. 8. Light output characteristic curves for five different phosphors.

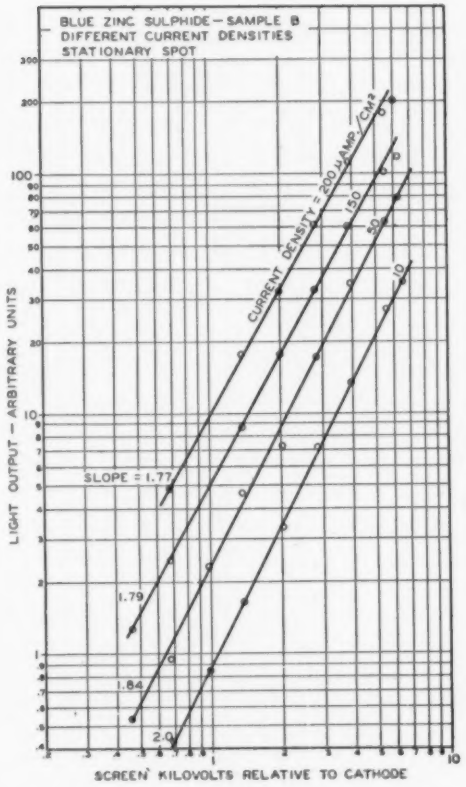

FIG. 9. Light output characteristic curves for a blue zinc sulphide bombarded at different current densities.

Other observers (W. B. Nottingham,[3] T. B. Brown[1]) have found that willemite has an exponent $n=2$ under some conditions, but measurements in this laboratory indicated that this was not, in general, the case. Consequently, an effort was made to determine the factors which might influence the exponent.

The most obvious factors involved are penetration of the material and probability of excitation of a center as a function of energy of bombarding electrons. If either or both of these factors could be changed by some treatment of the material, then it would be expected that the exponent of the voltage law would also change.

Conceivably, particle size might introduce secondary but important effects. Fig. 10 shows light output as a function of voltage for two

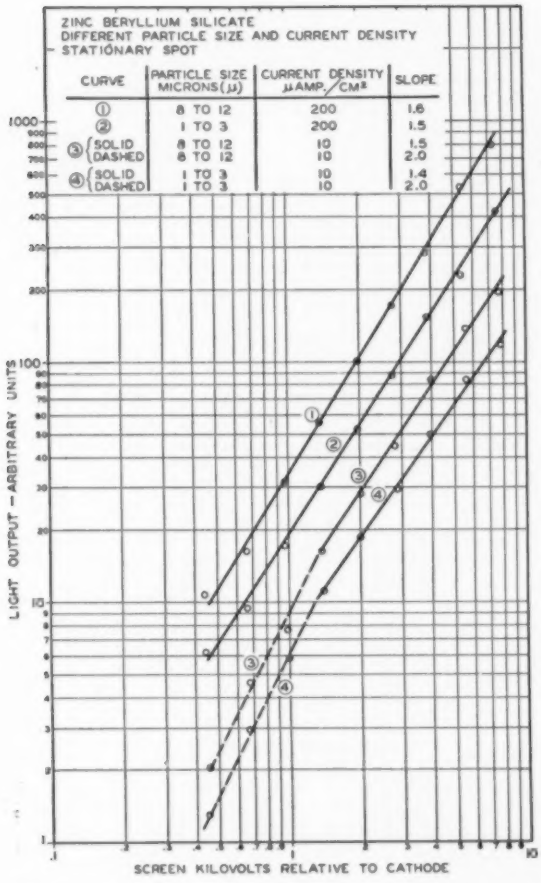
ZINC BERYLLIUM SILICATE
DIFFERENT PARTICLE SIZE AND CURRENT DENSITY
STATIONARY SPOT
CURVE | PARTICLE SIZE MICRONS (μ) | CURRENT DENSITY μAMP./CM² | SLOPE
① | 8 TO 12 | 200 | 1.6
② | 1 TO 3 | 200 | 1.5
③ SOLID | 8 TO 12 | 10 | 1.5
③ DASHED | 8 TO 12 | 10 | 2.0
④ SOLID | 1 TO 3 | 10 | 1.4
④ DASHED | 1 TO 3 | 10 | 2.0
LIGHT OUTPUT — ARBITRARY UNITS
SCREEN KILOVOLTS RELATIVE TO CATHODE

FIG. 10. Light output characteristic curves for zinc beryllium silicate screens of two different particle sizes and two different current densities. Screens were contained in the same envelope and material came from the same manufactured lot.

different particle sizes of zinc beryllium silicate, each taken at two widely different values of current density. Both screens were in the same envelope. At the lower current density, the two materials show nearly identical characteristics, with a fairly well-defined break at 1400 volts. Below this value, the materials seem to follow a voltage law with exponent two, and above it a law with somewhat smaller exponent. The upper lines are for the same two materials except that the current density is 200 microamperes per square centimeter. The break in each has disappeared, while the curve for the smaller particle size material has a somewhat smaller slope. The break disappears for values of current density greater than 20 microamperes per square centimeter.

The material of each different particle size range was obtained from the same lot of zinc beryllium silicate which was ground in an agate mortar to break down the lumps to single particles. The two groups of particles, each of a given size range, were separated out by settling in acetone. From this test, it may be concluded that there is no marked influence of the particle size on the voltage characteristics of the material, although this conclusion does not rule out the possibility of a penetration effect in very small crystals.

Curve 1 of Fig. 11 shows the light *versus* voltage characteristics of willemite after it had been fused in air to a platinum backing piece. This was done by heating the platinum (0.001-inch strip) with an electric current after sprinkling the powdered willemite on its upper surface. A platinum radiation shield was necessary to get complete fusion of the phosphor. On cooling, the platinum bent and no visible cracks appeared in the surface, although the appearance of the surface under bombardment shows, at low voltage, a couple of cracks running along the surface. These cannot be crystal boundaries because the higher voltages show that the light output between the cracks is quite non-uniform, which indicates the presence of a large number of small crystals.

It will be noted that the light *versus* voltage characteristics of this fused willemite sample show a definitely anomalous exponent which is much greater than that for any other material studied. However, this does not mean a greater over-all light output as might be inferred from the plot, since the absolute values of the ordinates of the separate curves bear no relation to each other. Curve 2 is for the same material which was fused in a graphite crucible, and then ground twenty-four hours in acetone. The real differences between the two samples are the following: the willemite in the fused layer had a greater opportunity to lose zinc from its surface than did the fused and ground willemite, the bombarded surface of the fused and ground sample had been fractured by grinding while that of the fused layer received no mechanical distortion, the fused and ground sample presented a rough surface composed of many small crystals to the

electron beam while the surface of the fused layer of willemite was quite smooth.

Curve 3, Fig. 11, is for willemite prepared from extremely pure materials by H. W. Leverenz, of the Victor Division, RCA Manufacturing Co., Inc. Its slope seems to be slightly less than that of the willemite prepared from ordinary chemically pure materials.

Analysis of Results

Suppose that a beam of electrons of constant cross section and constant current density, but variable energy, V, strikes a fluorescent material which is assumed to be homogeneous in structure. Suppose that the electrons lose energy continuously to the lattice as they pass through the material, and that the energy of an electron is determined (for a given initial energy at the surface) by its distance from the surface, and that this energy is represented by the function $U(x)$.

Distributed through the lattice of the crystal are thought to be active centers. These are much like free atoms in that they have an electron which may be excited to a higher energy level by the bombarding electrons, and which emits light when the electron returns to its ground state. The persistence of a phosphor is, of course, associated with the length of time the electron may stay in its excited state before returning to the ground state. It will be shown that the current-density characteristics of the light output may be largely explained by this phenomenon.

It is assumed that the rate of loss of energy with respect to distance (dU/dx) in traversing a region occupied by a center, is inappreciably greater than its rate of loss of energy through any other region of the material.

A slab of material having an area of one square centimeter and a thickness dx, is defined in the crystal such that its plane is perpendicular to the direction of the bombarding electrons, and its distance from the surface is x. If m is the fraction of the total area which is occupied by the active centers, and i (current density) is proportional to the rate of arrival of electrons at the slab, then the number of excitations per second for this slab is

$$dL = Amf(i)S(U)dx,$$

where $S(U)$ is the probability that an electron of energy U, striking an unexcited center, will excite it.

It has been found by Nottingham[3] as well as by the authors, that the function $f(i)$, to be treated more fully later, is substantially independent of voltage. Thus, the total light is

$$L = mAf(i)\int_0^{x_{max}} S(U)dx.$$

Now, x_{max} is the maximum penetration of the electron, and so for a given material, is a function only of the initial energy V of the electron. The rate of loss of energy as a function of energy is a quantity fundamental to the material, and some theoretical treatment of its nature has

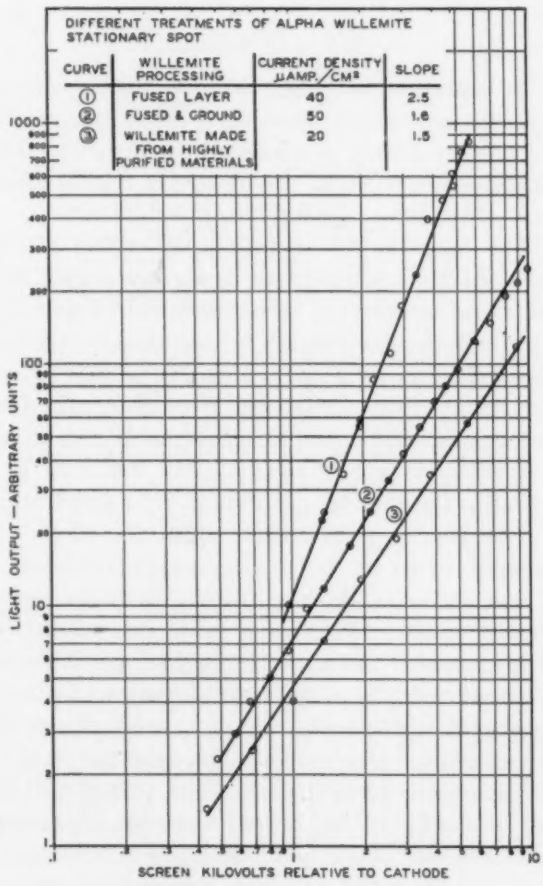
DIFFERENT TREATMENTS OF ALPHA WILLEMITE STATIONARY SPOT
CURVE
WILLEMITE PROCESSING
CURRENT DENSITY μAMP./CM²
SLOPE
① FUSED LAYER 40 2.5
② FUSED & GROUND 50 1.6
③ WILLEMITE MADE FROM HIGHLY PURIFIED MATERIALS 20 1.5
LIGHT OUTPUT — ARBITRARY UNITS
SCREEN KILOVOLTS RELATIVE TO CATHODE

Fig. 11. Light output characteristic curves for samples of willemite processed differently. Curve 3 presents data obtained from a screen of willemite made from extremely pure materials.

been given by Bethe.[7] Consequently, a simple transformation of the integral gives

$$L = Amf(i)\int_V^0 S(U)(dx/dU)dU.$$

The derivative of the light with respect to the lower limit gives just the integrand, or

$$dL/dV = -Amf(i)S(U)dx/dU|_{U=V}.$$

Now, experimentally $L \sim V^n$ and classically[7]

$$dU/dx \sim -1/U,$$

$$\therefore \left.\frac{dU}{dx}\right|_{U=V} = -\frac{1}{V}$$

so that $$S(V) \sim V^{(n-2)}.$$

This relation is of particular interest because the exponent for sulphides is almost exactly equal to 2. When n is equal to 2, we have

$$S(V) = \text{constant}.$$

This equation is based on the rather simple assumptions which have been made concerning the nature of the mechanism involved but the result serves to show the nature of the information which may be gained from such a study.

The function $f(i)$ represents the effect of the rate of arrival of electrons at a given region and has been assumed to be independent of voltage. This is borne out roughly by test on silicates but the test is not sufficiently accurate to show small variations. For the sample B of zinc sulphide, there does seem to be an appreciable change of $f(i)$ with voltage which is fairly uniform. For silicates, the change is too small to be significant.

The key to most of the behavior of $f(i)$ is thought to be in the persistence characteristics of the material. At low current densities (constant voltage), where the average rate of arrival of electrons is small, and the average interval between arrivals of successive electrons is large compared to the mean life of an excited center, the individual electrons may be expected to give independent effects and the light output will be proportional to the current density. However, when the rate of arrival of electrons is sufficiently great so that the interval between successive electrons is comparable to the mean life of the center in the excited state, then it is possible for some electrons to find the center excited by preceding electrons and no longer available for excitation. Under these conditions, the light output will fail to be linear with current density.

Suppose for simplicity that the center has only one excited state, and that the probability that it will be in this state at a time t after excitation, is $P(t)$. Then, the probability that the center will not be excited at the end of a time t after excitation is $1-P(t)$. Suppose the electrons which arrive at this center are strung out on a long line, one behind the other, so that they arrive at the center with a constant time interval, τ, between them, which is just the average interval between arrivals. Actually, the electrons will arrive with some random distribution in time, given by Poisson's law. However, it is commonly found in kinetic theory that the average of a random distribution may be used in an analysis and the form of the resulting expression will be the same as that found with a more sophisticated analysis except for a multiplicative constant. Consequently, it may be expected that this simple picture should lead to an approximation whose accuracy is sufficient to test other assumptions.

Suppose the center is excited at $t=0$, when the very first electron strikes it. The probability that the next electron (called electron 1) will also excite it, is just the probability that the center will be in the ground state, or not excited, at the end of the interval τ. We then have

$$P_1(e) = 1 - P(\tau).$$

If electron 1 does not excite it, what is the probability that the electron 2, coming at a time 2τ, or τ seconds after electron 1, will excite it? This is just the probability that the center will be still excited at the end of the first interval τ and will have returned to the unexcited state at the end of time 2τ. Since the return to ground state is pure chance, then if the center is observed to be excited at the end of any interval t, the probability that it will return to the ground state at the end of any succeeding interval s, is $1-P(s)$. The postulate that electron 1 did not excite the center is equivalent to an observation that the center is still excited at the time τ. Or, if the nth electron does not excite it, this is an observation that the center was still excited at the end of the interval $n\tau$.

[7] H. Bethe, Ann. d. Physik **5**, 325–400 (1930).

The probability that the center is still excited when electron 1 passes, is $P(\tau)$. The probability that it will have returned to its ground state at the end of the interval τ, following, is $1-P(\tau)$. The probability that both the one and the other will occur is their product or

$$P_2(e)=P(\tau)[1-P(\tau)].$$

Likewise, the probability that electron 3 will excite if electrons 1 and 2 have not, is

$$P_3(e)=P(2\tau)[1-P(\tau)].$$

Similarly for the nth electron

$$P_n(e)=P[(n-1)\tau][1-P(\tau)].$$

If a large number of electrons pass through the center, the fraction of all the excitations which occur with an interval $n\tau$, is proportional to $P_n(e)$. The total number of excitations will be proportional to the sum of the P_n's, where n assumes integral values from unity to some very large number (practically infinite).

Multiplying each fraction of the total number of excitations by the interval corresponding to this fraction, adding, and dividing by the total number of excitations gives the average time between excitations. The inverse of this is the average rate at which excitations occur. When the material has reached equilibrium under the action of the bombarding beam, the number of excitations must be equal to the number of de-excitations per unit time. This is just the rate of emission of quanta or the intensity of the emitted light, which is hereafter denoted by L.

Analytically

$$i=\frac{\tau P_1(e)+2\tau P_2(e)+3\tau P_3(e)+\cdots}{P_1(e)+P_2(e)+P_3(e)+\cdots}$$

$$=\frac{\tau[1-P(\tau)]+2\tau P(\tau)[1-P(\tau)]+3\tau P(2\tau)[1-P(\tau)]+\cdots}{[1-P(\tau)]+P(\tau)[1-P(\tau)]+P(2\tau)[1-P(\tau)]+\cdots}$$

$$=\frac{[1-P(\tau)]\sum_1^\infty n\tau P[(n-1)\tau]}{[1-P(\tau)]\sum_1^\infty P[(n-1)\tau]}$$

or

$$L=\frac{1}{i}=\frac{1}{\tau}\frac{\sum_1^\infty P[(n-1)\tau]}{\sum_1^\infty nP[(n-1)\tau]}.$$

The emission of a quantum of radiation is pure chance, therefore, as has been previously shown, the persistence characteristic for silicates, not sulphides, is given by $e^{-\alpha t}$ which may be expressed in the form

$$P[(n-1)\tau]=e^{-\alpha(n-1)\tau}=(e^{-\alpha\tau})^{n-1}.$$

Then,

$$L=\frac{1}{\tau}\frac{\sum_1^\infty (e^{-\alpha\tau})^{n-1}}{\sum_1^\infty n(e^{-\alpha\tau})^{n-1}}.$$

Let $e^{-\alpha\tau}=X$.

Then,

$$\sum_1^\infty X^{n-1}=1+X+X^2+X^3+\cdots=1/(1-X),\quad X<1$$

$$\sum_1^\infty nX^{n-1}=1+2X+3X^2+4X^3+\cdots[8]=1/(1-X)^2,\quad X<1$$

and

$$L=(1/\tau)(1-e^{-\alpha\tau}).$$

Now τ, the average interval between arrivals of electrons, is just $1/\beta$, where β is the average rate of arrival of electrons. Thus,

$$\tau=1/Bi$$

and

$$L=Bi(1-e^{-\gamma/Bi}).$$

B and γ are constants.

If there are two excited states in the same center, either one or the other of which may be excited, or if there are two sorts of centers in the material, each having its own persistence characteristic, then the decay characteristic will have the form given by curve A in Fig. 12,[9]

[8] B. O. Peirce, *Table of Integrals* (Ginn and Company, 1923), p. 89, formulae 755, 758.

[9] From data by T. B. Perkins, RCA Manufacturing Company.

which is willemite. Now each state, or each kind of center, will behave independently of the others and the light from each will be given by a formula of the preceding form. The total light will be the sum of the two. Specifically, the light from one type of center may be represented by

$$L_1 = Bia(1-e^{-\alpha/Bi})$$

and from the other center as

$$L_2 = Bib(1-e^{-\gamma/Bi}).$$

Then, the total light from both centers will be given by

$$L = L_1 + L_2 = Ai\left(1 \frac{-ae^{-\alpha/Bi}}{a+b} \frac{-be^{-\gamma/Bi}}{a+b}\right).$$

$$A = B(a+b)$$

From this equation it may be inferred that a plot of $\log(1-L/Ai)$ as a function of $1/i$ should have the same form as the persistence characteristic.

Figure 13 shows the persistence characteristic of zinc beryllium silicate analyzed to show the double exponential involved.[9] This should be compared with Fig. 14 which is a plot of the quantity $\log(1-L/iA)$ as a function of $1/i$, at constant voltage, for the same material.

In these plots curve A is the representation of the experimental points. Curve B is the extrapolation of the linear portion, and C is the plot of the logarithm of their difference. In all cases it is a surprisingly good straight line. The ratio of the slopes of the lines C and B should be the same in Figs. 13 and 14. There is a discrepancy of about 15 percent which is not surprising in view of the difficulties in making accurate measurements of the persistence characteristic.

Figure 15 is a plot of the quantity $\log(1-L/iA)$ as a function of $1/i$, made from data by W. B. Nottingham[3] on willemite. These data are the mean of all measurements made from 500 to 10,000 volts. Again the curve may be reduced to the sum of two exponentials, and the ratio of the slope of C to that of B is in very good agreement with the persistence characteristic as given by Fig. 12. Comparison of Fig. 13 with Fig. 12 serves to show how dependent the shape of the persistence characteristic is on the composition. Zinc beryllium silicate has a similar structure to that of willemite (zinc orthosilicate) and the same activator, except that about 10 percent of the zinc atoms are replaced by beryllium.

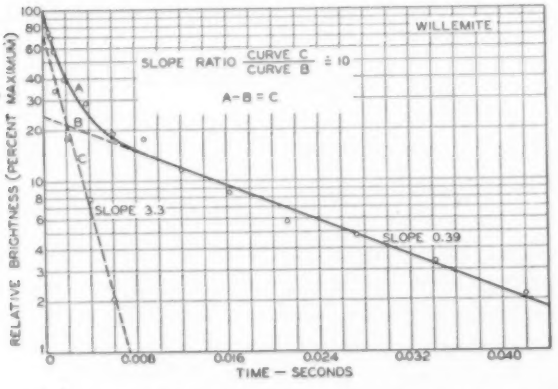

FIG. 12. Persistence characteristic of a sample of green willemite. Curve A is from plotted data. Extrapolating the straight-line portion backward and subtracting from A yields the curve C. This shows that the persistence may be fairly well represented as the sum of two exponentials.

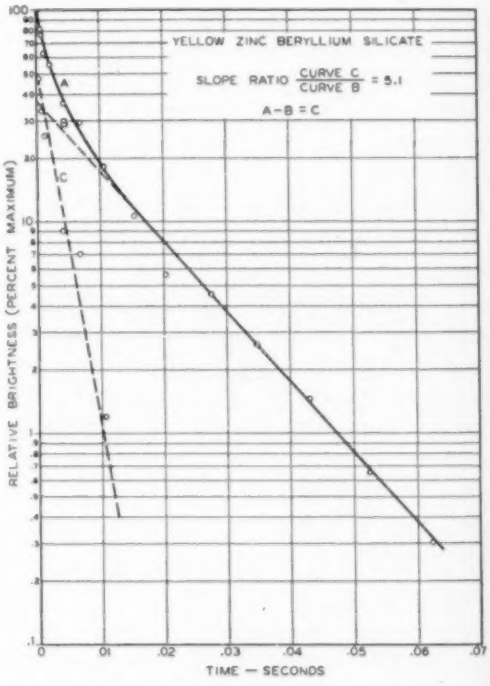

FIG. 13. Persistence characteristic of a sample of yellow zinc beryllium silicate analyzed to show two exponentials similar to Fig. 12.

Figure 16 is an analysis of data obtained on light output of willemite when the electron beam is completely scanning an area of 50 square

centimeters with the frequencies given on the plot. In view of the fact that in this case an electron spot having a diameter of 0.25 millimeter is moving across the screen with a velocity of about 2×10^5 cm/sec., the calculations which

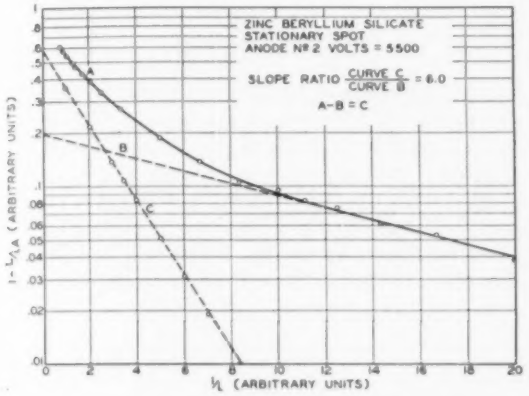

FIG. 14. Curves obtained as a result of an analysis of the light output data for zinc beryllium silicate.

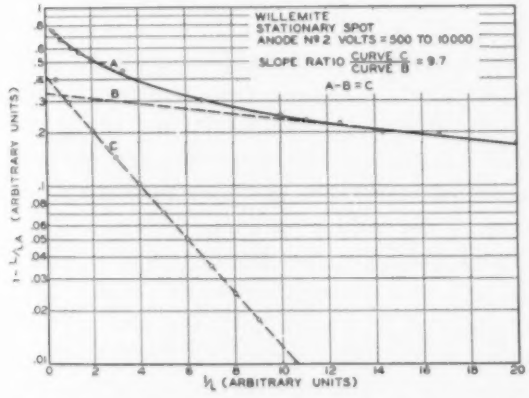

FIG. 15. Curves obtained as a result of an analysis of the light output data from Nottingham[3] on willemite. These values represent a mean of the data over the voltage range from 500 to 10,000 volts.

refer to an equilibrium state of bombardment, could hardly be expected to apply here. However, the fact that the quantity $(1-L/iA)$ may be resolved into two exponentials with argument $1/i$ is of interest.

To determine the quantity $(1-L/iA)$, it is necessary to plot log L against log i. The lowest part of the curve will be a straight line with slope 1. Extrapolating this, which is just iA,

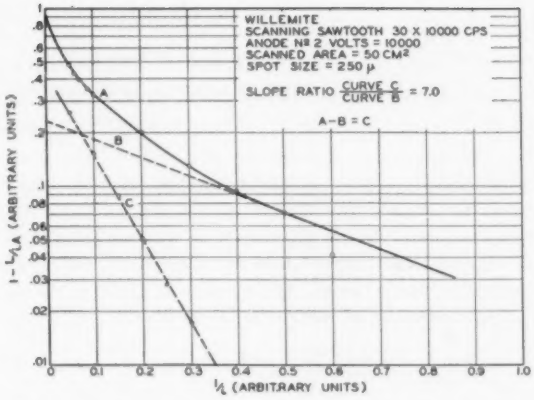

FIG. 16. Curves obtained as a result of an analysis of the light output data for a scanned spot on a willemite screen.

subtracting the value of L at a given value of i, and dividing by iA gives the desired quantity. At small values of i, where the curve first departs from linearity, it is easy to make a large error in this quantity as the difference is small. Thus, the points at large values of $(1/i)$, on the semi-log plot, are subject to appreciable error. On the other hand, the points at large values of i, very small $(1/i)$, are subject to error from experimental difficulties.

It is obvious from the plots that (experimentally):

$$\lim_{i\to\infty}(1-L/iA)\neq 1,$$

although this value is indicated by the analysis. Whether this discrepancy lies in the theory or the experiment cannot be ascertained at this time.

ACKNOWLEDGMENT

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Nerve Conduction with Distributed Capacitance

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Rashevsky's theory of nerve conduction has been extended to include the effects of distributed capacitance along the fiber. The simple theory of Rashevsky, based on the idea of local re-excitation by bioelectric currents according to Blair's law of excitation, leads to a formula for the initial velocity of the nerve impulse:

$$v=\frac{ak}{\log A_0/(A_0-\mathfrak{R})},$$

where a is the distance between Ranvier nodes, k is related to the chronaxie of the nerve, A_0, is the local exciting current, and $\mathfrak{R}$ is the threshold for excitation. This result does not depend explicitly on G, the node leakance. The present analysis gives for the initial velocity, in case $G/C \gg k$,

$$v=\frac{ak}{\log A_0/(A_0-(\mathfrak{R}+A_0\psi))},$$

where ψ is a complicated function of C and vanishes when $C=0$. In case G/C is not $\gg k$, the velocity is given as the root of a transcendental equation which may be solved graphically. Formulae for the asymptotic velocity of propagation are similarly related in Rashevsky's and the present case. Empirical data necessary to check the validity of these results are as yet unavailable; the type of experiments needed to obtain these data are suggested.

THE propagation of impulses in nerve has received considerable attention from biophysicists in recent years; in particular, a theory of conduction has been developed mathematically by N. Rashevsky[1] which has been shown, by H. A. Blair,[2] to be at least consistent with available electrical data on nerves. Rashevsky's theory, however, does not take into consideration the effects of distributed capacitance along the nerve fibers. In the hope of establishing a theory which is better adapted to the actual structure of nerve fibers, we propose in this paper to extend Rashevsky's results to include this complicating factor.

The theory of Rashevsky, from which we take our departure, may be briefly summarized as follows: A myelinated fiber is assumed to consist of a tube of fairly good conducting plasma surrounded by a poorly conducting myelin sheath, the sheath being broken at fairly regular intervals (nodes of Ranvier, Fig. 1). This tubular structure is supposed to be immersed in a medium of high conductivity. Excitation occurs at a given node along the nerve when a certain quantity p (its exact nature is left unspecified) exceeds a certain threshold h. The time variation of p at the nth node is assumed to follow Blair's equation[3]

$$dp_n/dt=KI-kp_n, \tag{1}$$

where I is the current flowing through the node in question, and K and k are constants. When a given node is excited, a source of e.m.f. appears at that node; this "action" e.m.f. gives rise to local bioelectric currents through the other, unexcited nodes, the intensity of the bioelectric currents depending on their distance from the excited node. The I in Blair's equation is identified with this bioelectric current, and by determining how it varies with distance along the nerve, Rashevsky derived a formula for the velocity of propagation of the nerve impulse in terms of electrical constants of the nerve. The formula is

$$v=\frac{ak}{\log (A_0-\beta\mathfrak{R})/(A_0-\mathfrak{R})}, \tag{2}$$

where a is the internodal distance, A_0 is the action current in the internodal interval immediately adjacent to the excited node, $\mathfrak{R}$ is the rheobase of the fiber (i.e., the minimum current required to excite), and β is an attenuation factor which for most nerves is generally small.

The problem outlined above of calculating the velocity of a nerve impulse falls naturally into

[1] N. Rashevsky, *Mathematical Biophysics* (University of Chicago Press, 1938), Chapters XIX, XX. This work will be referred to as *R*. Also see: N. Rashevsky, Physics **4**, 345 (1933).
[2] H. A. Blair, J. Gen. Physiol. **18**, 125 (1934).
[3] Reference 1, Chapter XVI.

two parts: first, the determination of the distribution of currents along the nerve in response to a suddenly established voltage at the zeroth node, and second, the description of the propagation of the excitation in accordance with Blair's equation. The distribution problem is essentially one of the type treated in the theory of electrical transmission lines,[4] and we shall consider it first.

Distribution of Currents in a Nerve

A nerve model of the type we have described can be conveniently replaced by an electrical network called an "artificial leaky cable" in electrical circuit theory. To do this we make the simplifying assumptions: 1. The external medium is a perfect conductor. 2. The distributed capacitance, C per internodal interval, is localized at each node. 3. The transverse leakances, G, of the nodes, are all the same. With these simplifications, the originally postulated nerve model is electrically equivalent to the lumpy line of Fig. 2, with longitudinal impedances z_1 consisting of the longitudinal resistance R of each interval, and transverse impedances z_2 made up of the transverse capacitances, C, and the leakances, G, in parallel. For convenience we shall label the

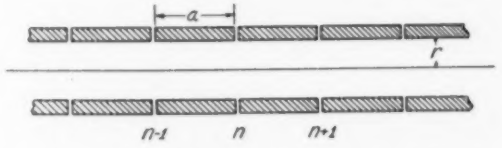

Fig. 1. From N. Rashevsky, *Mathematical Biophysics. Physico-Mathematical Foundations of Biology* (University of Chicago Press, 1938), p. 193. Reproduced by permission of the publisher.

interval next to the excited node the zeroth branch, the next one the first branch, and so on.

The problem to be solved is the following: if a steady unit e.m.f. is suddenly impressed at time $t=0$ in the zeroth branch of this network, what will be the resulting current, A_n, in the nth branch? (A_n is usually called the indicial admittance of the nth branch.)

[4] J. R. Carson, *Electric Circuit Theory and Operational Calculus* (McGraw-Hill, 1926). This work will be referred to as *C*.

If the initial longitudinal impedance were $\frac{1}{2}R$ instead of R (i.e., if the termination were "mid-series"), the solution would be comparatively simple. Using the results of Carson[5] for the nonleaky cable, and applying a well-known theorem from the operational calculus,[6] we find for this case,

$$A_n(t)=\frac{2}{R}\left[e^{-(2/RC+\lambda)t}I_n\left(\frac{2t}{RC}\right)+\lambda\int_0^t e^{-(\lambda+2/RC)\xi}I_n\left(\frac{2\xi}{RC}\right)d\xi\right], \quad (3)$$

where $\lambda=G/C$ and I_n is the Bessel function of imaginary argument and order n. Unfortunately,

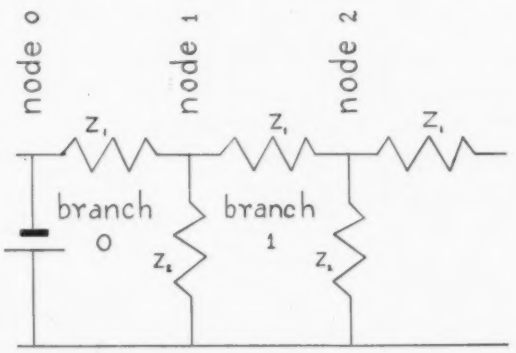

Fig. 2.

the termination we are interested in is not mid-series, since the initial impedance is R; the solution for this case cannot be given in closed form,[7] and so we shall attack the problem from a different point of view.

Artificial lines of the type we are considering are in many cases good approximations to actual lines in which the electrical constants of the line are continuously distributed. Hence if the branches of the artificial line are close enough together, the distribution in a continuous line, when suitably modified, should give a good approximation to the lumpy distribution. The problem of the continuous, leaky line is treated in full by Carson;[8] he gives for the indicial admittance at the point x along the cable

[5] Reference 4, p. 123.
[6] Reference 4, p. 45, theorem V.
[7] Reference 4, p. 143.
[8] Reference 4, Chapter VI.

$$A(x,t)=Z\left(\frac{e^{-((x^2RC/4t)+\lambda t)}}{\sqrt{t}}+\lambda\int_0^t\frac{e^{-((x^2RC/4\xi)+\lambda\xi)}}{\sqrt{\xi}}d\xi\right). \quad (4)$$

where the constants R, C, and G refer to unit length, and $Z=(C/\pi R)^{\frac{1}{2}}$. To apply this result to our problem we must replace xR by nR and xC by nC, the R and C in the lumpy case being per internodal interval. The required distribution can then be written

$$A(n,t)=Z\left(\frac{e^{-((n^2RC/4t)+\lambda t)}}{\sqrt{t}}+\lambda\int_0^t\frac{e^{-((n^2RC/4\xi)+\lambda\xi)}}{\sqrt{\xi}}d\xi\right). \quad (5)$$

For future reference we shall give the value of $A(0,t)$; i.e., the current acting in the zeroth branch or entering the first node. This is found from (5) by putting $n=0$:

$$A(0,t)=Z\left[\frac{e^{-\lambda t}}{\sqrt{t}}+\lambda\Phi(\lambda,t)\right], \quad (6)$$

where $\Phi(\lambda,t)$ is the error function,

$$\Phi(\lambda,t)=\int_0^t\frac{e^{-\lambda\xi}}{\sqrt{\xi}}d\xi.$$

Equation (5) shows how the current in the nth branch approaches its steady-state value after a unit voltage has been impressed on the line. The final steady-state current is found from (5) by putting $t=\infty$ and evaluating the definite integral. The result is

$$A(n,\infty)=\left(\frac{G}{R}\right)^{\frac{1}{2}}e^{-n(RG)^{\frac{1}{2}}}, \quad (7)$$

for a unit impressed e.m.f. In general, if A_0 is the steady current in the zeroth branch, the steady current in any other branch is

$$A(n,\infty)=A_0e^{-n(RG)^{\frac{1}{2}}} \quad (8)$$

and, from the definition of Z, $Z(\pi\lambda)^{\frac{1}{2}}=A_0$. Eq. (7) shows that the effect of excitation at one node falls off exponentially with the distance to that node. Thus, the experimental work of Erlanger and Gasser[9] indicates that the action current is quite negligible three or four nodes past the excited node. If this is so, then the propagation phenomenon may, in first approximation, be described by considering only that current at the nth node due to the $(n-1)$st excited one.[10] A better approximation, in which we consider the state of affairs two nodes from an excited one, will also be given for completeness.

Before treating the problem of excitation at the second node, we must first make some assumption as to how the zeroth node recovers after the excitation has jumped from the zeroth to the first node. In his original work, Rashevsky implicitly assumes that as soon as the $(n+1)$st node has been excited, the e.m.f. at the nth node is cut out of the circuit by what amounts to opening a switch. Strictly speaking, if our results are to be the logical generalization of Rashevsky's we should also cut out the zeroth e.m.f. by breaking the circuit at the zeroth node; however, again the mathematical problem is incapable of a closed solution, and so we shall have to approximate this recovery mechanism with one which is practically equivalent for our purposes but is much simpler to handle. This simplified recovery mechanism is the following: that after the excitation has passed the zeroth node, the e.m.f. in that node is short-circuited. Since the new battery (at the first node) is separated from the short circuit by the finite longitudinal resistance, R, the effect of the short circuit on the rest of the line will not be very different from that of an open circuit; in fact, we can with good approximation treat the distribution problem for an e.m.f. at the 1st node exactly as we did for the zeroth node excited.

Mathematically,[11] short-circuiting a unit e.m.f. in the zeroth branch at time t_1 (t_1 being the time necessary for the excitation to jump from the zeroth to the first node) is equivalent to introducing in this zeroth branch a counter unit e.m.f. at time $t=t_1$. The current distribution, $A'(n,t)$ in the nth branch for $t>t_1$, due to the original e.m.f. at the zeroth node, will consequently be given by the difference of two terms of the type in Eq. (5), one with $t=0$ as zero

[9] J. Erlanger and H. Gasser, *Electrical Signs of Nervous Activity* (University of Pennsylvania Press, 1937), p. 122.
[10] N. Rashevsky, Protoplasma **20**, 42 (1933).
[11] Reference 4, p. 90.

time, the other with $t=t_1$ as zero time:

$$A'(n,t)=Z\Bigg(\frac{e^{-((n^2RC/4t)+\lambda t)}}{\sqrt{t}}+\lambda\int_0^t\frac{e^{-((n^2RC/4\xi)+\lambda\xi)}}{\sqrt{\xi}}d\xi\Bigg)-Z\Bigg(\frac{e^{-[(n^2RC/4(t-t_1))+\lambda(t-t_1)]}}{(t-t_1)^{\frac{1}{2}}}+\lambda\int_0^{t-t_1}\frac{e^{-((n^2RC/4\xi)+\lambda\xi)}}{\sqrt{\xi}}d\xi\Bigg) \quad (9)$$

$$=Z\Bigg(\frac{e^{-((n^2RC/4t)+\lambda t)}}{\sqrt{t}}-\frac{e^{-[(n^2RC/4(t-t_1))+\lambda(t-t_1)]}}{(t-t_1)^{\frac{1}{2}}}+\lambda\int_{t-t_1}^t\frac{e^{-((n^2RC/4\xi)+\lambda\xi)}}{\sqrt{\xi}}d\xi\Bigg).$$

From Eq. (9) we see that $A'(n,t)=0$ for $t=\infty$; in other words, short-circuiting the battery in the zeroth node does not cause the current in the other branches to fall to zero immediately, but rather causes the current to decay at a rate depending on the capacitance and the resistance of the line. This is exactly the same phenomenon as the blurring of dots and dashes in rapid telegraphic signaling.

Because the current does not decay instantaneously, the bioelectric current in the first branch during $t>t_1$ will consist of two parts: first (and most important), the current due to the e.m.f. introduced at the first node at time $t=t_1$; and second (considerably smaller because of the space damping of Eq. (7)), the residual currents of Eq. (9) due to the previously excited zeroth node. We thus have for the total exciting current in the first branch during the time $t>t_1$,

$$A(1,t)=Z\Bigg[\frac{e^{-((RC/4t)+\lambda t)}}{\sqrt{t}}-\frac{e^{-[(RC/4(t-t_1))+\lambda(t-t_1)]}}{(t-t_1)^{\frac{1}{2}}}+\lambda\int_{t-t_1}^t\frac{e^{-((RC/4\xi)+\lambda\xi)}}{\sqrt{\xi}}d\xi+\frac{e^{-\lambda(t-t_1)}}{(t-t_1)^{\frac{1}{2}}}+\lambda\Phi(\lambda,t-t_1)\Bigg]. \quad (10)$$

Propagation of the Impulse

Having calculated the currents acting at the first and second nodes, we are now in a position to calculate t_1 and t_2-t_1, the time intervals required for the excitation to jump from the zeroth to the first node and from the first to the second node, respectively.

In order to determine t_1, we must integrate Blair's Eq. (1) with $I=A(0,t)$; this will give p_1 as a function of t, and t_1 will be specified by requiring that $p_1(t_1)=h$, where h is the threshold. Blair's equation governing the variation of p_1 at the first node may be written

$$\frac{dp_1}{dt}=KZ\Bigg[\frac{e^{-\lambda t}}{\sqrt{t}}+\lambda\Phi(\lambda,t)\Bigg]-kp_1; \quad (11)$$

its solution which satisfies the initial condition $p_1(0)=0$ is

$$p_1(t)=ZKe^{-kt}\times\Bigg[\Phi(\lambda-k,t)+\lambda\int_0^t\int_0^\tau\frac{e^{k\tau-\lambda\xi}}{\sqrt{\xi}}d\xi d\tau\Bigg]. \quad (12)$$

If we interchange the order of integration in the double integral in (12), we find

$$p_1(t)=ZKe^{-kt}\Bigg[\frac{k-\lambda}{k}\Phi(\lambda-k,t)+\frac{\lambda e^{kt}}{k}\Phi(\lambda,t)\Bigg]. \quad (13)$$

The time t_1 at which the first node becomes excited is determined by putting $p_1(t_1)$ in (13) equal to h. The resulting equation may be written in terms of the rheobase, $\mathfrak{R}$, and the action current A_0 since[12] $\mathfrak{R}=kh/K$, and, from the definition of Z, $Z(\pi\lambda)^{\frac{1}{2}}=A_0$:

$$\frac{\mathfrak{R}(\pi\lambda)^{\frac{1}{2}}}{A_0}=e^{-kt_1}[(k-\lambda)\Phi(\lambda-k,t_1)]+[\lambda\Phi(\lambda,t_1)]. \quad (15)$$

In general, to solve this equation for t_1, numerical or graphical methods must be employed. Probably the simplest scheme is to plot the two curves

$$y=\mathfrak{R}(\pi\lambda)^{\frac{1}{2}}-\lambda A_0\Phi(\lambda,t),\qquad y=A_0e^{-kt}(k-\lambda)\Phi(\lambda-k,t); \quad (16)$$

[12] Reference 1, p. 152, Eq. (12).

the value of t at which these two curves intersect is the required solution.

If $\lambda > k$, Eq. (15) can be simplified, since in this case

$$\Phi(\lambda-k, t)=\int_0^t \frac{e^{-(\lambda-k)\xi}}{\sqrt{\xi}}d\xi = \left(\frac{\pi}{\lambda-k}\right)^{\frac{1}{2}} - \int_t^\infty \frac{e^{-(\lambda-k)\xi}}{\sqrt{\xi}}d\xi \quad (17)$$

and so

$$\mathfrak{R}=A_0\left(1-\left(\frac{\lambda-k}{\lambda}\right)^{\frac{1}{2}}e^{-kt_1}\right) - \frac{A_0}{(\pi\lambda)^{\frac{1}{2}}}\left[\lambda\int_{t_1}^\infty \frac{e^{-\lambda\xi}}{\sqrt{\xi}}d\xi - (\lambda-k)e^{-kt_1}\int_{t_1}^\infty \frac{e^{-(\lambda-k)\xi}}{\sqrt{\xi}}d\xi\right]. \quad (18)$$

Finally, in case $\lambda \gg k$ (this would be true for very small C; i.e., those cases in which transient effects due to the capacitance of the fiber die down quickly as compared with the rise of p), we can neglect k in $\lambda-k$. Eq. (18) then becomes

$$\mathfrak{R}=A_0(1-e^{-kt_1}) - A_0\left(\frac{\lambda}{\pi}\right)^{\frac{1}{2}}\int_{t_1}^\infty \frac{e^{-\lambda\xi}}{\sqrt{\xi}}d\xi(1-e^{-kt_1}). \quad (19)$$

In this form we can clearly see the effect of the distributed capacitance on the time variation of p_1. The first part of (19) is the same as in the no-capacitance case,[13] while the integral is a term introduced by the capacitance which tends to slow down the rise of p_1. As we have pointed out, in this case ($\lambda \gg k$), this integral term will be of the nature of a small correction. We can therefore solve (19) for t_1 by successive approximations as follows:

Let t_1' be the root of

$$\mathfrak{R}=A_0(1-e^{-kt_1'}); \quad (20)$$

that is,

$$t_1'=\frac{1}{k}\log\frac{A_0}{A_0-\mathfrak{R}}. \quad (21)$$

[13] Reference 1, p. 152, Eq. (13). The use of the voltage, V, instead of A_0 involves a change in the constant K.

It is clear that t_1' will be a fair approximation to t_1. We can get a better second approximation by putting $t_1=t_1'$ wherever it appears in the second term of (19), and then solving again for t_1. This time we find

$$t_1=\frac{1}{k}\log\frac{A_0}{A_0-[\mathfrak{R}+A_0\psi(\lambda, t_1')]}, \quad (22)$$

where $\psi(\lambda, t_1')$ stands for

$$\psi(\lambda, t_1')=\left(\frac{\lambda}{\pi}\right)^{\frac{1}{2}}\int_{t_1'}^\infty \frac{e^{-\lambda\xi}}{\sqrt{\xi}}d\xi(1-e^{-kt_1'}). \quad (23)$$

The connection between the present development and Rashevsky's no-capacitance case is contained in Eqs. (21) and (22). Eq. (21) is exactly the expression given by Rashevsky for the time of the first jump; the more complicated (22) reduces to (21) for $C=0$ since $\lim_{C=0}\psi=0$.

The calculation of t_2-t_1 presents no particular difficulties. We first find the value of p_2 at the second node at time t_1 by solving Blair's equation with $A(1, t)$ from Eq. (5) as the exciting current, and $p_2(0)=0$ as the initial condition. The procedure is exactly the same as before, and we shall simply quote the result:

$$p_2(t_1)=\frac{A_0}{(\pi\lambda)^{\frac{1}{2}}}\frac{h}{\mathfrak{R}}\mu(t_1), \quad (24)$$

where

$$\mu(t_1)=e^{-kt_1}\left[(k-\lambda)\int_0^{t_1}\frac{e^{-[(RC/4\xi)+(\lambda-k)\xi]}}{\sqrt{\xi}}d\xi + \lambda e^{kt_1}\int_0^{t_1}\frac{e^{-((RC/4\xi)+\lambda\xi)}}{\sqrt{\xi}}d\xi\right].$$

Again, if $\lambda \gg k$, this may be written

$$p_2(t_1)=\frac{A_0}{\mathfrak{R}}e^{-(RG)^{\frac{1}{2}}}h\gamma(C, t_1), \quad (25)$$

where

$$\gamma(C, t_1)=(1-e^{-kt_1}) - \left(\frac{\lambda}{\pi}\right)^{\frac{1}{2}}\int_{t_1}^\infty \frac{e^{-((RC/4\xi)+\lambda\xi)}}{\sqrt{\xi}}d\xi(1-e^{-kt_1})e^{(RG)^{\frac{1}{2}}} \quad (26)$$

and in the limit $C=0$,

$$\lim_{C=0}\gamma(C,t_1)=1-e^{-kt_1}=\Re/A_0 \qquad (27)$$

by (19).

During the time t_2-t_1, the current acting at the second node is given by (10); hence we must solve Blair's equation for this time interval with $A(1,t)$ from (10) for the I of Blair's equation, and with the initial condition $p_2(t_1)=(A_0/\Re(\pi\lambda)^{\frac{1}{2}})h\mu(t_1)$. The result in this case is

$$p_2=\frac{A_0}{\Re(\pi\lambda)^{\frac{1}{2}}}h\mu(t_1)e^{-k(t-t_1)} +ZKe^{-kt}(A+B+C+D+E), \qquad (28)$$

where

$$A=\int_{t_1}^{t}\frac{e^{-((RC/4\xi)+(\lambda-k)\xi)}}{\sqrt{\xi}}d\xi$$

$$B=-\int_{t_1}^{t}\frac{e^{-((RC/4(\xi-t_1))+\lambda(\xi-t_1)-k\xi)}}{(\xi-t_1)^{\frac{1}{2}}}d\xi$$

$$C=e^{kt_1}\Phi(\lambda-k,t-t_1)$$

$$D=\lambda\int_{t_1}^{t}\int_{\tau-t_1}^{\tau}\frac{e^{-((RC/4\xi)+\lambda\xi-k\tau)}}{\sqrt{\xi}}d\xi d\tau$$

$$E=\frac{\lambda e^{kt}}{k}\Phi(\lambda,t-t_1)-\frac{\lambda e^{kt_1}}{k}\Phi(\lambda-k,t-t_1)$$

and t_2 is determined as the root of the equation obtained by setting $p_2(t_2)=h$. Again the calculation must be done numerically, or graphically; however, as before, if $\lambda\gg k$, the procedure simplifies considerably. To effect this simplification we note that in (28) only C and E are contributed by the currents due to the e.m.f. at the first node: A, B, and D are contributed by the e.m.f. at the zeroth node. In view of what we have already said about the rapid fall of current density along the line, A, B, and D will be considerably smaller than C and E; consequently, we can solve (28) by successive approximations in which A, B, and D are neglected in the first approximation. To proceed with the approximate solution it will again be convenient to transform (28) into a form involving the rheobase instead of the threshold by using the relation $\Re=kh/K$.

We thus have

$$\frac{\Re(\pi\lambda)^{\frac{1}{2}}}{A_0}=\mu(t_1)e^{-k(t_2-t_1)}+(k-\lambda)e^{-k(t_2-t_1)} \times\Phi(\lambda-k,t_2-t_1)+\lambda\Phi(\lambda,t_2-t_1) +ke^{-kt_2}(A+B+D), \qquad (29)$$

while if $\lambda\gg k$, this becomes

$$\frac{\Re(\pi\lambda)^{\frac{1}{2}}}{A_0}=(\pi\lambda)^{\frac{1}{2}}\gamma(t_1)e^{-(RG)^{\frac{1}{2}}}e^{-k(t_2-t_1)} +(\pi\lambda)^{\frac{1}{2}}(1-e^{-k(t_2-t_1)}) -\lambda\int_{t_2-t_1}^{\infty}\frac{e^{-\lambda\xi}}{\sqrt{\xi}}d\xi(1-e^{-k(t_2-t_1)}) +ke^{-kt_2}(A+B+D). \qquad (30)$$

Neglecting A, B, D, and

$$\int_{t_2-t_1}^{\infty}\frac{e^{-\lambda\xi}}{\sqrt{\xi}}d\xi,$$

in first approximation we find

$$t_2'=t_1+\frac{1}{k}\log\frac{A_0-A_0e^{-(RG)^{\frac{1}{2}}}\gamma(C,t_1)}{A_0-\Re}; \qquad (31)$$

if we substitute this value of t_2' wherever it appears in the second-order terms in (30), and solve for t_2-t_1 again, we obtain

$$t_2-t_1=\frac{1}{k}\log\frac{A_0-\gamma(C,t_1)e^{-(RG)^{\frac{1}{2}}}}{A_0-[\Re+A_0\psi_2(\lambda,t_2')]}, \qquad (32)$$

where

$$\psi_2(\lambda,t_2')=\lambda\int_{t_2'-t_1}^{\infty}\frac{e^{-\lambda\xi}}{\sqrt{\xi}}d\xi(1-e^{-k(t_2'-t_1)}) -ke^{-kt_2'}(A+B+D)$$

the limits in A, B, and D, being t_1 and t_2'. As before, (32) reduces to Rashevsky's result[14] when C approaches zero, since

$$\lim_{C=0}\gamma(C,t_1)=\Re/A_0 \quad\text{and}\quad \lim_{C=0}\psi=0.$$

In principle, the calculation outlined in the preceding paragraphs can be extended to calculate the general jumping time t_n-t_{n-1}, even

[14] Reference 1, p. 199.

without the assumption that the action current is very small two nodes away from the excited node. We have actually carried out this calculation but find that the result has the same general form as (29) or (30), (in case $\lambda \gg k$), except that $\gamma(C, t)$ and $(A+B+D)$ contain more terms of small size. Since the available experimental data seem to justify the assumption that the jumping process is fairly adequately explained by considering only the immediate neighborhood of the excited nodes, and since the general case presents no new mathematical steps, we shall not include those results in this paper. In any case, Eq. (22), or better, Eq. (32) should be sufficiently accurate to give an indication of the orders of magnitude involved.

Discussion of Results

If the internodal distance is a, the velocity of the nerve impulse will be (Eq. (22)), in case $\lambda \gg k$,

$$v = \frac{a}{t_1} = ak \Big/ \log \frac{A_0}{A_0 - [\Re + A_0 \psi_1(\lambda, t_1')]} \tag{33}$$

for the first jump, or

$$v = \frac{a}{t_2 - t_1} = ak \Big/ \log \frac{A_0 - \gamma(C, t_1) e^{-(RG)t_1}}{A_0 - [\Re + A_0 \psi_2(\lambda, t_2')]} \tag{34}$$

for the second jump. A direct comparison of (33) or (34) with experiment is at present impossible because of the paucity of consistent data all taken at the same time on a single fiber.

Qualitatively, Eq. (34) predicts certain relations which are actually found to hold experimentally. Thus, from (34) the velocity would be expected to increase with a, the internodal distance, and k, which is found to be true, at least for small velocities.[2] There are considerable data available on the variation of velocity with fiber diameter, all of it indicating the the velocity is proportional to the diameter raised to a power lying between one and two. It has been suggested that this relationship is really one between fiber diameter and internodal distance, internodal distance being an approximately linear function of diameter.[15] On the basis of (34) we should expect a qualitative relation between v and the diameter of the required sort, inasmuch as k is probably a monotonic increasing function of a, at least for small velocities. However, until data are available on the exact dependence of k on fiber diameter or internodal distance, such predictions must remain conjectural.

It will be interesting to note exactly what data are necessary in order to make an adequate comparison of (33), (34) or, (in case λ is not $\gg k$), (29), with experiment. Values for A_0, and $\Re$, the action current and rheobase per fiber, have been derived by H. A. Blair from time-lag excitation data.[2] The values quoted, however, are open to some question since the time lags, which were of the order of a millisecond, were only estimated but not measured. Inasmuch as Blair's values are entirely different from those calculated by Rashevsky,[16] a further experimental study of these time lags would be very useful.

The impedance measurements of Cole[17] are of a type which should eventually yield values of the other structural electric constants. For the distributed capacity, C, there is little doubt that the values run around $1 \mu f/cm^2$. On the other hand, there seem to be no consistent data for R, the internodal resistance, and G, the nodal leakance.

It is unfortunate that more specific quantitative checks with experiment are not possible because of the present lack of data; but it is hoped that at some future time the constants used in this analysis will be measured on a single preparation with a view to checking the predictions of this theory.

Acknowledgment

The writer takes pleasure in acknowledging the assistance given him by Professors Carl Eckart and Nicolas Rashevsky in the preparation of this paper. Also, he thanks Dr. A. S. Householder for checking the algebraic details. The problem was suggested by Dr. Rashevsky.

[15] R. W. Gerard, Quart. Rev. Biol. **6**, 59 (1931).

[16] N. Rashevsky, Physics **4**, 345 (1933).

[17] K. S. Cole and H. J. Curtis, Cold Spring Harbor Symposium on Quantitative Biology **4**, 89 (1936).

Problem of Ice Formation

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A solution of the nonlinear problem of the rate of ice formation on a long cylinder whose surface temperature is variable (but always below 0°C) is given. The first-order correction to the steady-state solution is evaluated and found to be small. As an example, application is made to a specific problem in freezing soil.

THE following problem has application to the frequent engineering procedure of stabilizing wet soil by freezing. A cylinder of infinite length and radius r_1 is imbedded in water which is initially at the constant temperature of 0°C. Find the temperature distribution outside the cylinder if the surface of the cylinder is maintained at the temperature $-C(t)$, this temperature being always below 0°C. As the cooling progresses, a shell of ice will be formed between r_1 and a variable radius r_2, where the temperature is 0°C. Outside r_2 the temperature retains its initial value of 0°C and the flux of heat there remains zero. The controlling factor in the above problem, and the one which distinguishes it from ordinary problems in heat conduction, is the liberation of latent heat at r_2 and the establishment of an ice-region between r_1 and r_2 where the conductivity is different from that of water.

In the ice region, the temperature, T, satisfies

$$\frac{\partial T}{\partial t}=k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right), \tag{1}$$

where k is the coefficient of diffusivity of ice. Let the boundary of the ice at time t be at r_2 and at time $t+dt$ at r_2+dr_2. Then in a cylindrical shell of one cm length and of radii r_2 and r_2+dr_2 there will be liberated during dt a quantity of heat equal to $L\rho_2 2\pi r_2 dr_2$, where L is the latent heat of fusion and ρ_2 is the density of water. Since there is no flow of heat across r_2+dr_2, the above quantity of heat must equal $K2\pi r_2(\partial T/\partial r)dt$, where K is the conductivity of ice. Hence we must have at r_2

$$\partial T/\partial r=\gamma(dr/dt), \quad (\gamma=L\rho_1/K) \tag{2}$$

and

$$T=0. \tag{3}$$

In the (r, t) plane the curves of constant temperature are given by

$$T(r, t)=C. \tag{4}$$

Along these lines we have

$$(\partial T/\partial t)dt+(\partial T/\partial r)dr=0,$$

$$(dr/dt)=-(\partial T/\partial t)/(\partial T/\partial r). \tag{5}$$

When applied to the line of $T=0$°C, (5) determines dr_2/dt. Hence (2) is equivalent to

$$(\partial T/\partial r)^2=-\gamma(\partial T/\partial t). \tag{6}$$

The problem is therefore *nonlinear*[1] and a general solution cannot be obtained by superposition of particular solutions. The circumstance which makes the problem manageable and also suggests a method of solution is the fact that the heat capacity of the ice is small in comparison to the latent heat of freezing; hence for moderate rates of cooling at the source the temperature-distribution in the ice-region approximates to the steady state which would obtain if r_2 were fixed. This method was used by J. Stefan[2] in his solution of the one-dimensional analog of our problem. Guided by this idea, we attempt a solution of (1) of the following form:

$$T=A+B\ln x+\left(\frac{r_1^2}{k}\right)[\tfrac{1}{4}(A-\dot{B})x^2+\tfrac{1}{4}\dot{B}x^2\ln x]$$

$$+\left(\frac{r_1^4}{k^2}\right)\left[\frac{x^4}{128}(2\ddot{A}-3\ddot{B})+\frac{\dddot{B}}{64}x^4\ln x\right]+\cdots, \tag{7}$$

where $x=(r/r_1)$, A and B are functions of time

[1] The statement in the 7th edition of B. Riemann and H. Weber's *Differential-Gleichungen* (Fried, Wieweg and Sons, Braunschweig, 1927), Vol. II, p. 222, that the difficulty of this problem arises from its nonhomogeneity is evidently in error.

[2] J. Stefan, Sits. Akad. Wiss. Wien **98**, 965 (1890).

and the dots denote differentiation with respect to time. It can be verified by direct substitution that the form (7) satisfies (1), and its usefulness will depend upon the rapidity of convergence, if indeed it converges at all. By assumption, the

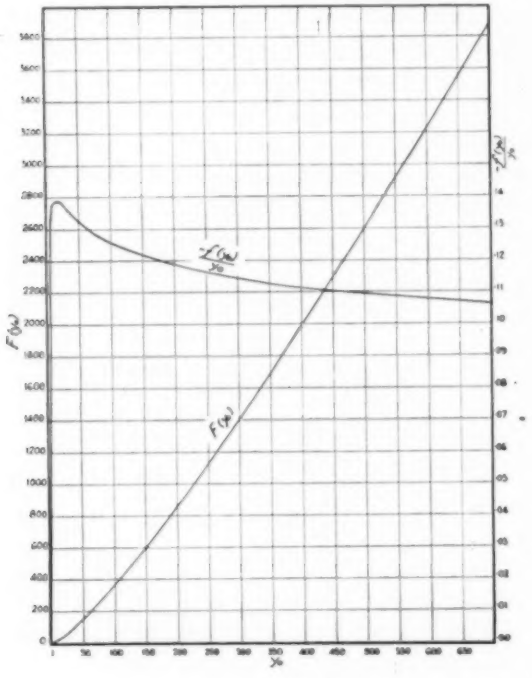

FIG. 1. $F(y_0)\equiv y_0 \ln y_0+1-y_0$, $\dfrac{f(y_0)}{y_0}\equiv\dfrac{2(y_0-1)}{y_0(\ln y_0)^2}-\dfrac{(1+y_0)}{y_0 \ln y_0}$, $y_0\equiv(r_2/r_1)^2$.

solution (7) is to hold only for $t>0$, and for $1\leqslant x\leqslant r_2/r_1$; while for larger values of x, $T=0$. Initially, at $t=0$,

$$T=-C, \quad \text{for} \quad x=1 \tag{8}$$

$$T=0 \qquad \text{for} \quad x>1. \tag{9}$$

We now proceed to determine A and B from the initial conditions and the boundary conditions at r_1 and r_2. It follows from Eqs. (7) and (8) that

$$-C=A+\frac{r_1^2}{4k}(A-\dot{B})+\frac{r_1^4}{128k^2}(2\ddot{A}-3\ddot{B})+\cdots, \tag{10}$$

while Eqs. (7) and (9) yield

$$A+\tfrac{1}{2}B \ln y+\frac{r_1^2}{k}\left[\tfrac{1}{4}(\dot{A}-\dot{B})+\frac{\dot{B}}{2}y \ln y\right]+\cdots=0. \tag{11}$$

Also, from (7) and (2):

$$2B+\frac{r_1^2}{k}\left[y(A-\dot{B})+\frac{\dot{B}}{2}(y \ln y+y)\right]+\cdots=r_1^2\gamma\frac{dy}{dt}, \tag{12}$$

where $y=(r_2/r_1)^2$. With C given, Eqs. (10), (11) and (12) serve to determine A, B, and y as functions of time. For these we attempt solutions of the following form

$$A=A_0+\left(\frac{r_1^2}{k}\right)A_1+\left(\frac{r_1^4}{k^2}\right)A_2+\cdots, \tag{13}$$

$$B=B_0+\left(\frac{r_1^2}{k}\right)B_1+\left(\frac{r_1^4}{k^2}\right)B_2+\cdots, \tag{14}$$

$$y=y_0+\left(\frac{r_1^2}{k}\right)y_1+\left(\frac{r_1^4}{k^2}\right)y_2+\cdots. \tag{15}$$

When substituted in (10), (11) and (12), the terms independent of (r_1^2/k) yield the relations

$$A_0=-C=-\tfrac{1}{2}B_0 \ln y_0, \quad 2B_0=r_1^2\gamma\dot{y}_0. \tag{16}$$

Hence

$$\ln y_0 dy_0=\frac{4C}{r_1^2\gamma}dt, \tag{17}$$

for which we choose the solution

$$y_0 \ln y_0+1-y_0=\frac{4}{r_1^2\gamma}\int_0^t C dt. \tag{18}$$

Insofar as y_0 is a sufficient approximation to y, and this will be shown to be the case for all but extremely large absolute values of C, (18) shows that y depends solely on the time integral of the cooling at the cylinder.

Now y_1 can be determined from the (r_1^2/k) terms in (10), (11) and (12). These yield

$$A_1+\tfrac{1}{4}(A_0-\dot{B}_0)=0, \tag{19}$$

$$A_1+\tfrac{1}{2}B_1 \ln y_0+\tfrac{1}{2}B_0\frac{y_1}{y_0}+\frac{y_0}{4}(A_0-\dot{B}_0)+\frac{\dot{B}_0}{8}y_0 \ln y_0=0, \tag{20}$$

$$2B_1+y_0(\dot{A}_0-\dot{B}_0)+\frac{\dot{B}_0}{2}(y_0\ln y_0+y_0)=r_1^2\gamma\dot{y}_1. \quad (21)$$

On eliminating A_1 and B_1 from the above, one obtains a first-order differential equation for y_1:

$$\dot{y}_1+y_1\left(\frac{\dot{y}_0}{y_0\ln y_0}\right)=\frac{1}{2r_1^2\gamma}\dot{B}_0y_0\ln y_0 + \frac{1}{r_1^2\gamma}(A_0-\dot{B}_0)\left[y_0-\frac{(y_0-1)}{\ln y_0}\right], \quad (22)$$

or

$$\frac{dy_1}{dy_0}+\frac{y_1}{y_0\ln y_0}=\frac{1}{2r_1^2\gamma}y_0\ln y_0\frac{dB_0}{dy_0} + \frac{1}{r_1^2\gamma}\left(\frac{dA_0}{dy_0}-\frac{dB_0}{dy_0}\right)\left[y_0-\frac{(y_0-1)}{\ln y_0}\right]. \quad (23)$$

With the aid of Eqs. (16) and (17), (23) reduces to:

$$\frac{dy_1}{dy_0}+\frac{y_1}{y_0\ln y_0}=\frac{1}{r_1^2\gamma\ln y_0}\frac{d}{dy_0} \times\left\{C\left[-1-y_0+\frac{2(y_0-1)}{\ln y_0}\right]\right\}. \quad (24)$$

Now at $t=0$, $y=1$, and by Eq. (18), $y_0=1$. Therefore, in Eq. (15), $y_1=0$ when $y_0=1$, and the appropriate solution of (24) is

$$y_1=\frac{C}{r_1^2\gamma}f(y_0), \quad f(y_0)\equiv\frac{2(y_0-1)}{(\ln y_0)^2}-\frac{(1+y_0)}{\ln y_0},$$
$$f(1)\rightarrow-\tfrac{1}{6}(y_0-1).$$

Noting that

$$\gamma=L\rho_1/K=L\rho_1/k\rho_2C_2\simeq 2L/k,$$

where c_2 is the heat capacity of ice, we obtain from (15)

$$y=y_0+(\rho_2C_2/L\rho_1)Cf(y_0)+\cdots. \quad (25)$$

The first term in Eq. (25) furnishes, through Eq. (8), the steady-state solution for the case of r_2 fixed, and at temperature 0°C. The deviation from this temperature distribution, as represented by the second term, is proportional to the ratio of the specific heat of ice to its latent heat of fusion. The function $f(y_0)/y_0$ is shown in Fig. 1, and it is seen that it has a maximum value of only 0.139. Thus, with a moisture content of 30 percent, and a temperature amplitude of 25°C, the maximum error introduced by neglecting the second term in (25) is less than 7 percent. At 60 percent moisture content, the error is about 3 percent. Fig. 2 shows the growth of the ice cylinder (on the basis of the first term only in Eq. 25) for several assumed moisture contents of the soil. At r_1, which was assumed to be 5.72 cm (4″ standard pipe size), a temperature $-24\sin(\pi N/195)$°C is maintained, where N is the number of days after the beginning of the cooling.[3] The assigned value for K of 0.0045 is intermediate between the quoted values of 0.005 for ice and 0.0037 for slightly moist soil. In varying the moisture content, this value of K was kept constant while the latent heat of fusion was made proportional to the moisture content.

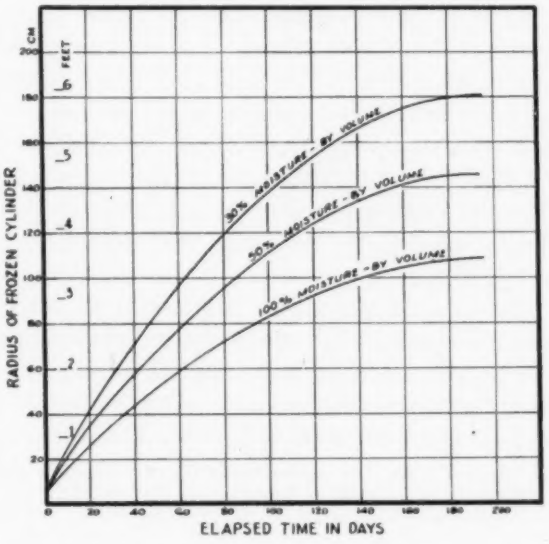

FIG. 2. Freezing of soil about a long pipe. Radius of pipe = 5.72 cm (4″ standard pipe size). Temperature of pipe (in °C) $=-24\sin(\pi N/195)$, where N = elapsed time in days from beginning of freezing, initial temperature of soil = 0°C.

[3] The data for Fig. 2 have reference to an application in open-pit mining in Alaska, suggested by W. J. Mead; the source of cooling is provided naturally, by winter temperatures. The temperature variation, $C=-24\sin(\pi N/195)$°C, assumed at the source is a close approximation to the actual mean temperature variation obtained during the freezing season.

Synthesis of a High Frequency Reactance

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FOSTER'S reactance theorem[1] provides the necessary information for the synthesis of the most general dissipationless reactance provided the frequency range is such that lumped reactances, namely inductance coils and condensers, are available. At the higher frequencies these are difficult to obtain practically and the impedance elements must be treated in design as the transmission lines with distributed inductance and capacity which they truly are. Accordingly, it is convenient to have general methods for the synthesis of a high frequency reactance presented in terms of the number, arrangement, and constants of the transmission line reactance elements. Two such methods applying to dissipationless reactances over a definite, limited frequency band, as most commonly needed in practice, are given in the following.

The most general pure reactance, no matter what the frequency may be, must according to the Foster theorem have the form shown by curve of reactance *versus* frequency of Fig. 1. Since only the form of the curve is intended to be pictured here, the abscissa might equally well have been δ instead of frequency, where δ is proportional to the deviation in frequency from some base frequency, say f_0, and is most conveniently defined by:

$$\delta=((f-f_0)/f_0)\pi/2.$$

Suppose that in a certain frequency band, say between $\delta=\delta_0$ to $\delta=\delta_{2n}$ where $1\gg\delta_{2n}\gg\delta_0$, a reactance is desired which shall have zeros at $\delta=\delta_1$, $\delta=\delta_3$, $\cdots$, $\delta=\delta_{2n-1}$ and infinities at $\delta=\delta_0$, $\delta=\delta_2$, $\cdots$, $\delta=\delta_{2n}$.

One form of function which will meet these specifications and also possess everywhere a positive slope as does the curve of Fig. 1 is

$$Z=jA\delta\frac{(\delta^2-\delta_1^2)(\delta^2-\delta_3^2)\cdots(\delta^2-\delta^2_{2n-1})}{(\delta^2-\delta_0^2)(\delta^2-\delta_2^2)\cdots(\delta^2-\delta^2_{2n})}, \quad (1)$$

in which A is a negative, real constant. Expanding by partial fractions

$$Z=jA\delta\left[\frac{B_0}{\delta^2-\delta_0^2}+\frac{B_2}{\delta^2-\delta_2^2}+\cdots+\frac{B_{2n}}{\delta^2-\delta^2_{2n}}\right], \quad (2)$$

in which

$$B_m=\frac{(\delta_m^2-\delta_1^2)(\delta_m^2-\delta_3^2)\cdots(\delta_m^2-\delta^2_{2n-1})}{(\delta_m^2-\delta_0^2)(\delta_m^2-\delta_2^2)\cdots(\delta_m^2-\delta^2_{m-2})(\delta_m^2-\delta^2_{m+2})\cdots(\delta_m^2-\delta^2_{2n})}$$

and is seen to be positive. Thus a group of impedances of the form $jAB_m\delta/(\delta^2-\delta_m^2)$, connected in series will give the desired reactance. One way to realize an impedance of this form is with two transmission lines, properly chosen and connected in parallel.

The impedance looking into a transmission line, shorted at its far end, of characteristic impedance Z_1, and one-quarter of a wave-length long at the frequency f_0 is[2]

$$Z_{in}'=jZ_1\tan\theta, \quad (3)$$

which is very nearly equal to $-j\dfrac{Z_1}{\delta}$.

$$\left(\theta=\frac{\pi}{2}\left[1+\frac{f-f_0}{f_0}\right]=\frac{\pi}{2}+\delta\right.$$

so that sin θ is very close to unity and cos θ is practically equal to $-\delta$ for small values of δ).

[1] R. M. Foster, "A Reactance Theorem," Bell Sys. Tech. J. 5, 259 (1924); or E. A. Guillemin, *Communication Networks* (John Wiley and Sons, 1935), Vol. II.

[2] See, for instance, W. E. Everitt, *Communication Engineering* (McGraw-Hill, 1937), Chap. V.

The impedance looking into a transmission line, open circuited at its far end,[3] of characteristic impedance Z_2, and one-quarter of a wavelength long at the frequency f_0 is

$$Z_{in}'' = jZ_2\delta. \tag{4}$$

The resultant impedance of the two lines in parallel is:

$$Z_m = -j\frac{\delta Z_1}{\delta^2 - Z_1/Z_2}. \tag{5}$$

So if $Z_{1,m}$ is made equal to $-AB_m$ and $Z_{2,m}$ made equal to $Z_{1,m}/\delta_m^2$, then n such parallel combinations in series will suffice to provide the desired reactance.

Since the choice of the zero and infinity points, δ_1, δ_2, etc., has been purely arbitrary, it is possible to employ the inverse network of that just obtained as a satisfying form for the needed reactance. The inverse network's reactance will have zeros wherever the previous network had infinities, and vice versa. The new network will consist of a group of n admittances connected in parallel[4] each of the form of Z_m in Eq. (5). It

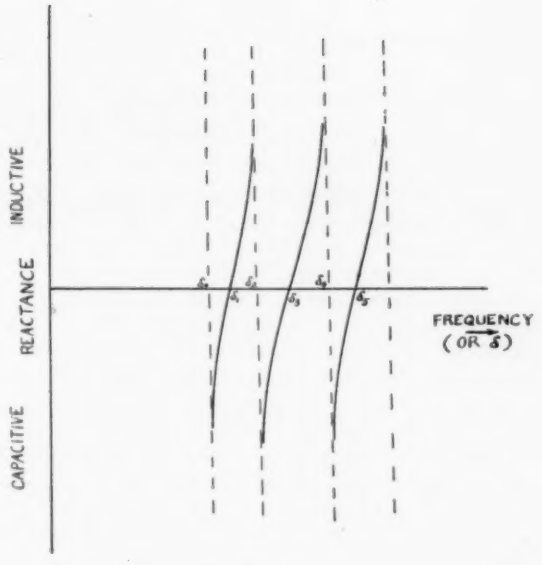

FIG. 1.

can easily be checked that an admittance of this form results from Z_{in}' and Z_{in}'' connected in series. Thus a group of n such series combinations connected in parallel will also give the desired reactance.

Both networks give reactances which are symmetrical about the point $\delta = 0$. In some cases this may be advantageous; in other cases it is simply necessary to place the operating frequency band entirely above or entirely below $f = f_0$.

[3] If it is undesirable to use an open-ended line, then a shorted, half-wave line may be used instead, for it has an input impedance of this same form.

[4] This by virtue of the well-known inverse impedance theorem which states that the inverse impedance of a group of impedance elements connected in series is obtained by connecting the inverses of the impedance elements in parallel.

Letters to the Editor

A High Frequency Amplifier and Oscillator

The very efficient high frequency resonators described in this Journal by Hansen[1] have been made the basis for the construction of amplifiers and oscillators of a new type in which the transit time of the electrons, which is usually considered a source of serious difficulty at very high frequencies, has been turned to constructive use.

Power is obtained from a stream of cathode rays at constant voltage and current. In the amplifier, these rays pass along the electric field of a Hansen resonator, or Rhumbatron, as it is now called, which is made to oscillate by the signal to be amplified. For best results, this Rhumbatron is of a type having its ends made re-entrant, so as to shorten its dimension along the field. The rays enter and leave the Rhumbatron through grids or holes, designed to minimize the ohmic losses and yet to confine the field practically all within the Rhumbatron. This field may be very weak, if the signal is weak, since it needs only to produce a slight modulation of the speeds of the cathode rays, but not to stop any of them nor even to change their speeds by more than a very moderate fraction of their speed at the entrance to the Rhumbatron.

At its exit, therefore, there is almost no modulation of the current carried by the cathode rays. Subsequently, however, in a much longer flight through a field-free space, the differences between the speeds of different electrons causes them to gather into bunches, separated by rarefactions. This bunching process changes the stream from pure d.c. to an equal d.c. plus a high frequency a.c. component.

By sending the stream in this condition through a second Rhumbatron, a part of its power can be transferred to oscillations of that Rhumbatron.

The first Rhumbatron, which causes the cathode rays to gather themselves into bunches, is called the "buncher," the Rhumbatron receiving their power is the "catcher," and the whole apparatus has been named a "Klystron."

If the buncher is driven by an external source of power, such as an antenna receiving radiation, and the cathode rays are strong enough to give the catcher more power than the antenna gives to the buncher, the Klystron is acting as an amplifier. If the buncher is driven by power received through a coupling loop or otherwise from the catcher, the Klystron is acting as an oscillator. And finally, if the buncher is driven by power from both of these sources at once, the Klystron is acting as a regenerative amplifier.

When the device acts as an oscillator, the proper phase relations between the bunches of cathode rays arriving at the catcher and the field therein, which must take power from them, can be secured by a proper choice of the transit time of any electron in the center of a bunch; and this time is governed by the potential difference between the cathode and the Rhumbatron.

Modulation is accomplished by changing either this potential or the cathode-ray current, or in ways related to these; and detection is accomplished most readily by using the difference in the speeds of the slowest electrons beyond the catcher, with and without the signal to be detected, or doing the same with the fastest electrons.

RUSSELL H. VARIAN
SIGURD F. VARIAN

Stanford University,
Stanford University, California,
January 6, 1939.

[1] W. W. Hansen, J. App. Phys. **9**, 654 (1938).

Simplified Calculation of the Arbitrary Constants of a Linear Differential Equation of the Second Order with Constant Coefficients

The differential equation

$$\frac{d^2y}{dx^2}+2a\frac{dy}{dx}+by=0, \tag{1}$$

where a and b are constants, may be solved by introducing either

$$y=\epsilon^{-ax}(A\cosh mx+(B/m)\sinh mx) \tag{2}$$

or

$$y=\epsilon^{-ax}(A\cos nx+(B/n)\sin nx) \tag{3}$$

with A and B as arbitrary constants and

$$m=(a^2-b)^{\frac{1}{2}} \tag{4}$$

and

$$n=(b-a^2)^{\frac{1}{2}}. \tag{5}$$

Eq. (2) represents the solution for $a^2>b$, and Eq. (3) is the solution for $a^2<b$. For $a^2=b$ either Eq. (2) or Eq. (3) will become with $m=n=0$:

$$y=\epsilon^{-ax}(A+Bx). \tag{6}$$

Taking either Eq. (2) or Eq. (3) as general solution to the given differential Eq. (1), the solutions for both the other cases may be found simply by varying the value of m or n, respectively.

Let y_0 and y_0' at $x=x_0$ be the initial conditions of the differential Eq. (1), the arbitrary constants may become

$$A=\epsilon^{ax_0}\left\{y_0\cosh mx_0-\frac{ay_0+y_0'}{m}\sinh mx_0\right\} \tag{7}$$

and

$$B=\epsilon^{ax_0}\{(ay_0+y_0')\cosh mx_0-y_0m\sinh mx_0\} \tag{8}$$

for $a^2>b$. The values for $a^2<b$ and $a^2=b$ may be calculated by varying the value of m accordingly.

R. FEINBERG

51 Tolcarne Drive,
Northwood Hills,
London, England,
January 3, 1939.

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EASTMAN SPECTROSCOPIC PLATES, Ultraviolet Sensitive, have recently been improved by the use of a new fluorescent ultraviolet sensitizing layer which has no granular structure. This should result in better definition in the images of spectral lines.

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Innovations in Instruments

Professional Kodachrome Film, Daylight Type

Professional Kodachrome Film is now available in a type accurately balanced for daylight use in a number of popular sizes up to and including 8×10 inches, the Eastman Kodak Company announces from Rochester, New York.

Identical in faithfulness of color reproduction with the Professional Kodachrome Film, Type B, recently announced for studio use, the new Professional Kodachrome Film, Daylight Type, extends the advantages of Kodachrome in large sizes to the professional and commercial photographer who wishes to make direct color photographs out-of-doors.

Professional Kodachrome Film can be used in ordinary cut film holders and can, therefore, be used in any camera accepting such holders. Any good anastigmat lens properly corrected for transverse and axial chromatic aberration—briefly, any lens which gives critically sharp definition everywhere in the field for panchromatic film—may be used satisfactorily.

When Professional Kodachrome Film, Daylight Type, is used in sunlight or light of equivalent color-temperature, no filter is required. Full-color transparencies are taken with a single brief exposure.

For this film, a Weston rating of 5 is recommended, and the average exposure in sunlight for an average subject is 1/25 second at *f*:6.3.

While the color balance of Professional Kodachrome Film, Daylight Type, is adjusted to produce correct rendering with average noon sunlight, without a filter, certain filters may be employed to compensate for variations in daylight color with different conditions of weather or subject. Need for compensation may arise, for example, when a subject is photographed in open shade, the sky then providing the chief illumination, at a color temperature higher than that of direct sunlight.

The filters which may be required under various daylight conditions are:

Situation	Wratten Filter for Sunlight Effect
1. Subject in open shade, very blue sky, no clouds	2A
2. Subject in open shade, blue sky, white clouds	2A or 1
3. Subject in open shade, light blue sky, white clouds	1
4. Subject illuminated by hazy sky	1 or none
5. Under overcast sky (varies considerably in color)	1 or none
6. Subject illuminated by direct sunlight	None

Professional Kodachrome Film, Daylight Type, has moderate contrast and fair latitude, so permitting some tolerance in exposure and normal contrast in the lighting of the subject. However, exposure must be more carefully calculated than with the black-and-white films commonly used, and lighting contrasts must not be extreme.

The color saturation and contrast of full-color transparencies made on Professional Kodachrome Film are such that satisfactory three-color prints can usually be made from them without employing masking methods. These transparencies are especially suitable as originals from which color-separation negatives can be made for the preparation of color prints on paper by the methods commonly used, and for photomechanical reproduction. Since the Kodachrome process is subtractive, transparencies have unusual clarity and brilliance, features especially valuable when the film is intended for direct viewing or for binding between glass for use as lantern slides.

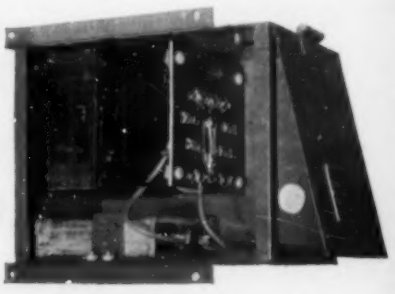
Professional Kodachrome Film, Daylight Type, is packaged in boxes each containing six films. Also included in each box is a return envelope, to be used when fewer than three films are returned for processing at one time; extra sheets of black paper for packing between exposed films, an instruction booklet, and a gummed address label.

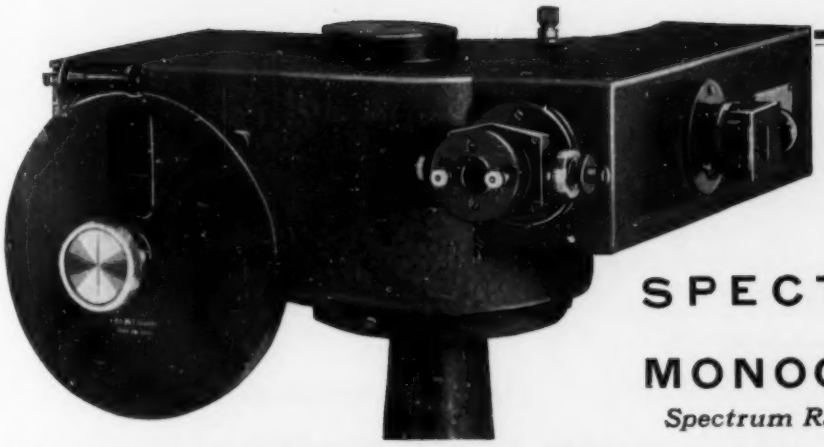
All films must be returned to Rochester for processing. When three or more films are returned at one time, there is no processing charge; when fewer than three films are returned at one time, a small service charge is made. The processed transparencies are returned to the photographer in protective sleeves of clear Kodapak, which need be removed only when the transparency is used for making separation negatives.

Sizes in which Professional Kodachrome Film, Daylight Type, is now available are: 2¼×3¼ inches, 6.5×9 cm, 9×12 cm, 3¼×4¼ inches, 4×5 inches, 5×7 inches and 8×10 inches.

Speed Controller for A.C. Motors

Illustrated is a Raytheon multistep speed controller for a.c. motors which makes it possible to vary the motor speed by changing the input voltage. This method of speed control permits the motor being operated by a thermostat or automatic switch because it definitely insures that the motor will always restart, regardless of the controller setting. A typical controller is illustrated showing its principal components, an autotransformer with voltage





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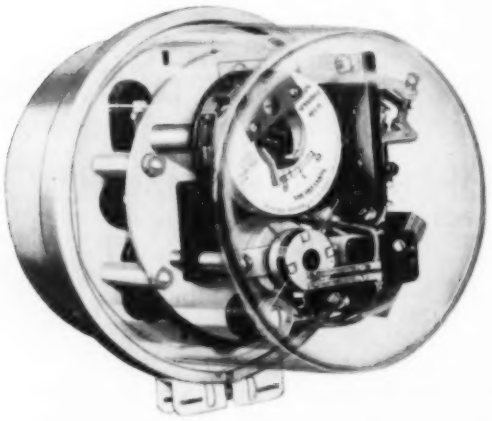
Write for complete information giving a.c. supply line voltage, frequency, number of phases, also motor nameplate data and desired input voltages. Address Raytheon Manufacturing Company, 156 Willow Street, Waltham, Massachusetts.

Ultra-Sensitive Relay

The Microtrol relay, housed in a weatherproof case, makes available for the first time a completely automatic, ultra-sensitive controller, entirely self-contained. The input relay (Model 705) will operate on values from two microamperes up and from 0.5 millivolt up. The power relay contacts will safely handle five amperes at 110 volts a.c. noninductive. Made by the Weston Electrical Instrument Corporation, Newark, New Jersey, this relay is widely applicable in various circuit functions such as the control of automatic doors, smoke alarm or liquid density control, a.c. and d.c. voltage or current alarms, and over- or under-temperature or thermocouple failure.

The control units—sensitive relay, power relay and clock motor—are housed within a plug-in type glass case, similar to the glass case used to enclose watt-hour-meters. Permitting removal of the working mechanism without disconnecting any wires, the front housing offers ease of maintenance; a spare can be quickly and easily inserted without interrupting service.

The Microtrol relay, combining the Sensitrol relay Model 705 and the power relay Model 712, is designed for an operating interval in which the power relay will open and close once each four seconds. The relay will remain closed approximately three seconds and open one second. These time intervals can be changed, on special order, so that the device will close once each 15 seconds, once each minute, or remain closed continuously until conditions return to normal.



Available in the same ranges as the Sensitrol relay Model 705, the Microtrol relay is intended for use on 120-volt, 60-cycle circuits. For special requirements, the relay can be supplied for any commercial frequency, at voltage ranges from 12 to 250 volts. Complete information about the new Microtrol Relay, Model 726, may be obtained by writing the Weston Electrical Instrument Corporation, Newark, New Jersey.

Electrical Developments of the Past Year

Use of Hydrogen in Motors to Reduce Friction

Hydrogen, because it is 10 times lighter than air, reduces the windage or friction of rotating shafts of large generators to one-tenth that in air. When a 130,000-pound steel rotor revolves, its outside rim travels at more than 750 miles an hour. The windage reduction caused by the insertion of hydrogen is a vital factor in increasing power efficiency. Thus generators may be operated efficiently at much higher speeds than previously.

The economy of space resulting from the high speed operation of generators is indicated by the fact that a 25,000 kilowatt generator operating its rotor at 1800 revolutions a minute weighs 16 pounds per kilowatt and occupies 21½ square feet of floor space, compared with only 7.3 pounds and 10.3 square feet in the same capacity generator operating at 3600 revolutions a minute.

New Transformer Design

A newly developed method of winding the cores of distribution transformers was commercially announced during the year. The magnetic circuit of previous transformers consisted of hundreds of hand-assembled laminations. The wound-core, or Spirakore, design permits cores to be machine-wound around the transformer coils from a continuous strip of cold-rolled steel in a fraction of the time required for hand assembly. Such construction has led to higher efficiency at heavy loads, better voltage regulation at the higher power-factors, low exciting current, lighter weight, and savings in cost that resulted in substantial price reductions.

Gas-Filled Cables

Gas-filled cable, offering numerous advantages over types of cable previously available, is another important development. Similar in general construction to the widely used oil-filled cable, the channel spaces are drained before the cable leaves the factory, and the cable system is filled with gas under a moderate pressure of approximately 10 pounds per square inch. For an equal life the insulation thickness may be reduced safely as compared with the solid-type cable of conventional design. Gas-filled cable is admirably fitted for installation in existing duct systems without enlargement of manholes; it can be handled in the

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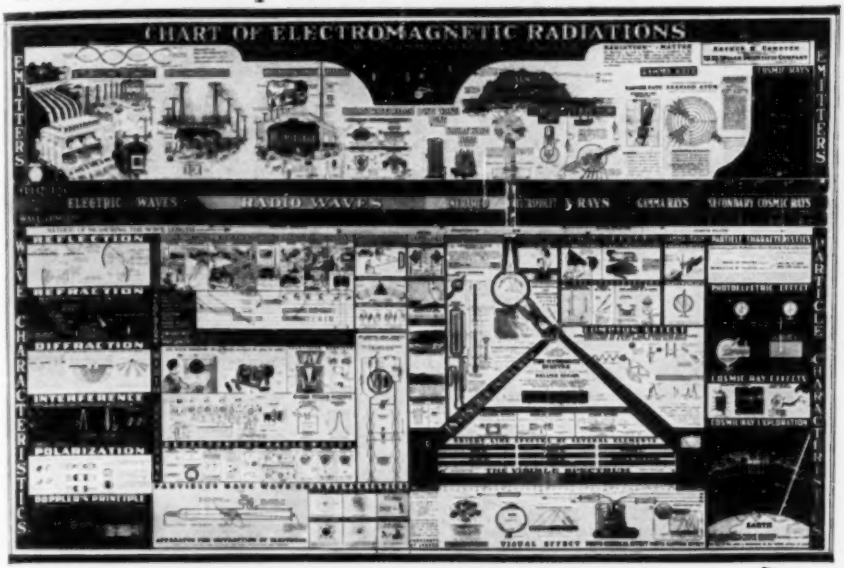

gives both the elementary and latest known facts associated with the complete wave and frequency scales.

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field almost as easily as solid-type cable; splices are simple and easily constructed; gas pressure may be controlled by simple reservoirs; and a simple alarm system may be installed to detect gas leakage.

Automatic Rheostats

Research engineers at the Westinghouse Research Laboratories have developed the "Silverstat" to keep powerful auxiliary generators from over-loading no matter how heavy or light their load. It is actually an automatic rheostat using silver buttons. This device, in its most compact form, takes up no more room than a cigar box, and its buttons are no larger than the polkadots on a man's tie, but it can maintain the voltage of generators within one or two percent of normal. The "Silverstat" includes a voltage-sensitive stationary coil energized by the generator to which it is attached. The coil is mounted on an iron magnetic circuit equipped with an air gap. A moving aluminum arm is so constructed that an iron armature on its lower end can move against the pull of a spring, depending on the magnetic field set up by the voltage in the stationary coil. Pulled by the magnet or forced by the spring, the top end of the aluminum arm closes and opens in succession a series of silver buttons, each mounted at the free end of an individual leaf spring. The silver buttons are wired in sequence from the fixed ends of their leaf springs to consecutive steps of a stationary regulating resistance. By compressing or releasing these springs, the amount of resistance is controlled automatically.

New Uses for "Electric Eyes"

"Electric Eyes" were busier this year than ever before, counting, sorting and controlling tedious operations. For example, a single phototube, amplifier and a small reversible motor were coordinated to take the kinks out of paper cutting. Light from a small lamp passes through four small circular lenses revolving 1800 times a minute on the motor's shaft. In operation, these four revolving light beams strike a roll of paper on which is printed a dark narrow guide line and are reflected back through the phototube. As long as the light beam strikes only the light surface of the paper roll, the resistance of the phototube remains uniform, but as soon as the beam intercepts the dark guide line, the tube's resistance changes. Two thyratron tubes amplify the electrical energy of the phototube and change the voltage of the motor, causing it to operate in the direction required to bring the paper roll back into line so that mechanical cutters can follow a straight line.

A similar phototube, or rather, batteries of them, went to work in a steel mill during the year, looking for holes. They keep watch on yard-wide ribbons of thin sheet steel, inspecting enough sheet in one minute to make 7000 cans for soup, milk and other liquids. This electric "hole finder" consists of a row of nine photoelectric tubes, an amplifier, thyratron control panel and a marking device. As the steel strip speeds past at 800 feet a minute, the "eye" detects and marks holes in the steel as small as one-hundredth of an inch; that's three times smaller than a pin hole.

Another tube, much larger than the phototube, made its bow to the steel industry during 1938. Known as an "Ignitron," it stands two feet high and is the newest development in mercury arc rectifiers to change alternating current into direct current to operate steel mill equipment. The "Ignitron" operates on the same principle as a little glass radio tube which controls the direction and amount of electrical waves entering a radio receiver. By means of an electric arc through the mercury vapor in a vacuum tube that is automatically turned on and off at every cycle (or change of direction) in the alternating current, electrons are allowed to flow only in one direction through the tube. They come out as direct, one-directional current.

Illumination

A new era in artificial lighting was initiated during the spring with the introduction of fluorescent Mazda lamps, considered by many the greatest single development of light production in many years. The spectral quality of light from the daylight lamp is the closest approach to natural daylight that it has ever been possible to produce by any artificial illuminant at an efficiency even approaching that of these lamps; and, similarly, the efficiencies with which these lamps produce colored light have never been approached previously. Colored light is produced in hitherto unobtainable pastel tints as well as in pure colors. The tubular lamps contain a small globule of mercury and a small amount of argon gas at low pressure, and preheated coiled tungsten wire electrodes covered with an electron-emissive material. The low-pressure mercury arcs produce ultraviolet radiation which activates the fluorescent chemicals, or phosphors, with which the inside surfaces of the tubes are coated. The phosphors are energy transformers—they step the invisible ultraviolet radiation down to the visible part of the spectrum. By proper selection and blending of the phosphors, it is possible to produce radiation in practically any desired part or parts of the spectrum. Fluorescent lamps, available only a few months, already have found applications in many different lighting fields and will be features of the coming World's Fairs.

A new mercury lamp surpasses in brilliancy most known illuminants; the arc attains a brightness of the order of 30,000 candles per sq. cm. Highest in wattage and smallest in size of all the mercury lamps, the new 1000-watt water-cooled lamp, with an efficiency of 65 lumens per watt, is a quartz capillary tube about three-inches long, containing a small amount of mercury and fitted at each end with a brass ferrule for electric contact. The arc stream itself is less than an inch long and scarcely larger in diameter than an ordinary pin. The intense heat generated by such an arc would melt even the quartz tube almost immediately if some effective method of removing the heat was not employed; the arc tube is operated in a rapidly moving stream of water within a glass water jacket. This new lamp promises to make an important place for itself in the fields of photoengraving and blueprinting. It requires less current than do carbon arcs used in photoengraving, produces superior work because of the steadiness of the arc, and makes it possible to produce clean cut engravings of poster

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size. For blueprinting it makes possible increased printing speeds, more compact equipment, and in some cases lower wattages. The lamp is also being considered as a light source for searchlights of certain types.

Also introduced during 1938 was the projector lamp, in which a reflector of highly accurate contour and a lens plate are actually parts of the lamp itself.

Sound System for the Golden Gate Exposition

A complex and extensive sound network is now being installed for the Golden Gate Exposition in San Francisco Bay, by RCA Victor engineers to entertain hundreds of thousands of visitors. The large sound reenforcement system utilizes a battery of approximately 60 powerful high fidelity loudspeakers of a type especially designed and built for use on the exposition grounds. Mounted in 36 locations, they will deliver 3000 watts of audiopower to fill the island with theme music, and to make announcements and redistribute important programs. The system provides for picking up programs from any one of twelve points on the grounds, and for distribution to any one or more of the 36 groups of loudspeakers. Six different programs may be handled through the system at one time. In addition, there is an automatic announcer system by means of which the entire sound installation can be thrown open for emergency announcements with the normal operating volume quadrupled by throwing a single switch. There is also provision for reproducing recorded music through the system to all parts of the grounds, and for recording broadcasts or programs on the grounds for reproduction through the system.

New Magnet Wires

Formex, a new insulated magnet wire, exhibits remarkable improvements in many characteristics. The resistance of its insulation to mechanical abuse such as stretching, bending and abrasion is consistently from two to five times greater than that of conventional magnet wires, and it retains its toughness and flexibility when heated at machine operating temperatures long after insulation on conventional magnet wire has become brittle.

The year witnessed extended use of fiber glass for insulating motor windings and for other applications. Wire with such insulation has good heat resistance, and, when properly treated with the right varnish or compounds, has good mechanical and electrical properties. The insulation is flexible, has high dielectric strength, and, because of the special varnish, is moisture- and abrasion-resistant. Motors subject to severe operating conditions, such as severe dust or dirt conditions and high operating temperatures, were made with such insulation.

Sterilamp

Another type of lamp using invisible bactericidal radiation in place of light performed new health services during 1938. Known as a "Sterilamp," this device radiates ultraviolet rays which kill bacteria.

Besides sterilizing the atmosphere in hospital operating rooms, killing germs on glasses and eating utensils in restaurants, the "Sterilamp" went into commercial use in meat markets and packing concerns. Because of its ability to kill mold and bacteria in meat, the new lamp permits an increase in the refrigerator temperature from the usual 38 degrees Fahrenheit to as much as 50 degrees, reducing dehydration of the meat and at the same time cutting down on refrigeration costs.

Spectrographic Assays of Vitamins

For the first time in the history of medicine, standards depending on analysis with the spectrograph were accepted last year by the council on pharmacy and chemistry of the American Medical Association. Dr. Henry R. Kreider of the A.M.A.'s chemical laboratory related the spectroscopic requirements for riboflavin, or vitamin B_2. Standards for medicinal products have heretofore been determined largely by chemical and physical methods. Now Dr. Kreider says the spectroscope provides an excellent means of standardization.

Ultraviolet absorption spectra has been found particularly adapted to the precise analysis of vitamins. Dr. A. E. Ruehle, Bell Telephone Laboratories, has made extensive use of this method to secure hints as to what products were formed in various chemical reactions with vitamin B_1, how the atoms in the vitamin molecule divided, and to confirm chemical findings.

The vitamin molecule, it was found, could be chemically split into two parts and by comparing the spectra of one of these and its derivatives with those of the corresponding derivatives of a chemical known as thiazole, strong evidence was obtained that there was a so-called thiazole ring in the vitamin. This was later confirmed by chemical synthesis, leading to artificial manufacture of the vitamin. Investigation has shown that the spectroscope is the best method of assaying vitamin A. And evidence was obtained with the spectroscope which affords reason to believe there is more than one kind of vitamin A—possibly several massive groupings of molecules, all very similar, and each carrying the medicinal or physiological effects of vitamin A. With remarkable precision the spectroscope shows the existence of these apparently different groups, extremely close together.

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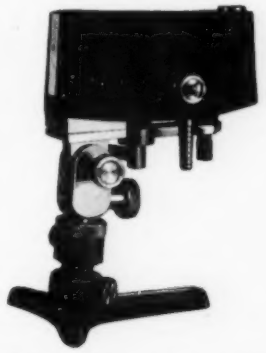

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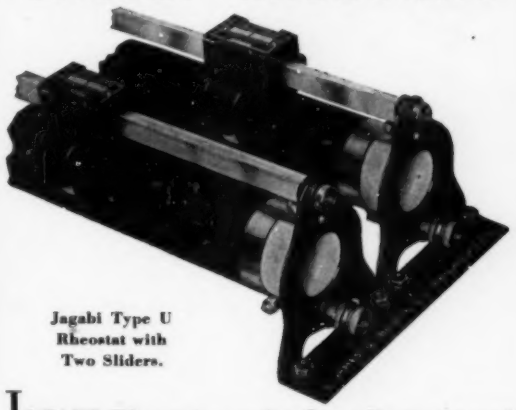

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